ARTICLE

TONTINE PENSIONS

JONATHAN BARRY FORMAN† & MICHAEL J. SABIN††

INTRODUCTION .................................................................................. 757

I. PENSIONS, ANNUITIES, AND OTHER LIFETIME INCOME MECHANISMS TODAY ............................................................... 761
   A. Social Security ............................................................................ 761
   B. Pensions .................................................................................... 763
      1. Retirement Savings Are Tax-Favored.................................. 763
      2. Types of Pension Plans ...................................................... 764
      3. The Regulation of Employment-Based Plans ...................... 768

† Alfred P. Murrah Professor of Law, University of Oklahoma; B.A. 1973, Northwestern University; M.A. (Psychology) 1975, University of Iowa; J.D. 1978, University of Michigan; M.A. (Economics) 1983, George Washington University; Professor in Residence at the Internal Revenue Service Office of Chief Counsel, Washington, D.C. for the 2009-2010 academic year; Member of the Board of Trustees of the Oklahoma Public Employees Retirement System, 2003-2011.
†† Independent consultant, Sunnyvale, CA; B.S. (Electrical Engineering) 1977, University of Florida; M.S. 1979, Ph.D 1984 (Electrical Engineering), Stanford University; Member of Technical Staff, Bell Laboratories, 1977-1981; Assistant Professor (EECS), University of California Berkeley, 1984-1986.

C. Other Sources of Lifetime Income .................................................. 769
   1. Systematic Withdrawals ...................................................... 770
   2. Lifetime Annuities ............................................................ 772
   3. Longevity Insurance ......................................................... 772
   4. Other Lifetime Income Products ......................................... 773

II. Tontine Pensions .................................................................. 774
A. The Tontine Principle .......................................................... 774
B. A Tontine Fund ................................................................... 775
   1. A Fair Transfer–plan ......................................................... 776
   2. Expected Benefits of Tontine Funds ................................... 784
   3. Two Problems with Tontine Funds ..................................... 786
C. A Tontine Annuity ............................................................... 790
   1. Monthly Accrual of Fair Transfer–plan Payouts .................. 790
   2. Annuity Payback .............................................................. 790
   3. Adding in Investment Income ............................................ 794
   4. Managing Investments ....................................................... 795
   5. Adverse Selection Is Always a Challenge for Annuities ..... 800
D. Tontine Pensions ................................................................. 802
   1. A Simple Tontine Pension .................................................. 803
   2. Tontine Pensions Compared with Other Pension Alternatives .................................................. 804
   3. Summary of the Advantages and Disadvantages of Tontine Pensions .................................................. 807

III. Modeling a Simple Tontine Pension ........................................ 808
A. The Parameters of the Simulation ......................................... 808
B. Calculation of the Retirement Balance ................................... 809
C. Calculation of the Monthly Tontine-Pension Distributions ....... 812
D. Adequacy ............................................................................. 813
E. Tontine Pensions in the Real World ........................................ 815

IV. Replacing the California State Teachers' Retirement System with a Tontine Pension ......................................... 815
A. Background on the California State Teachers' Retirement System .......... 816
B. Replacing the California State Teachers' Retirement System Defined Benefit Plan with a Tontine Pension ............ 817

V. Solving the Technical Problems of Creating a Tontine Pension ............................................................................. 818
A. Taxation of Benefits .............................................................. 818
B. Legal Issues ......................................................................... 818
C. Dealing with Market Volatility .............................................. 822
D. Gender Issues ........................................................................ 823
INTRODUCTION

Tontines are investment vehicles that can be used to provide retirement income. A tontine is a financial product that combines the features of an annuity and a lottery.¹ In a simple tontine, a group of investors pool their money together to buy a portfolio of investments and, as investors die, their shares are forfeited, with the entire fund going to the last surviving investor. Over the years, this “last survivor takes all” approach has made for some great fiction.² For example, in an episode of the popular television series *M*A*S*H*, Colonel Sherman T. Potter, as the last survivor of his World War I unit, got to open the bottle of French cognac that he and his buddies bought (and share it with his Korean War compatriots).³ On the other hand, sometimes the fictional plots involved nefarious characters trying to kill off the rest of the investors to “inherit” the fund.⁴


⁴ See, e.g., *The Simpsons*: Raging Abe Simpson and His Grumbling Grandson in “The Curse of the Flying Hellfish” (Fox television broadcast Apr. 28, 1996) (depicting an episode in which Grampa Simpson reveals to his grandson Bart that he and Montgomery Burns were part of a World War II American army unit that stole priceless art from a German castle, which the last surviving unit member will inherit); see also *The Wild Wild West*: The Night of the Tottering Tontine (CBS television broadcast Jan. 6, 1967) (portraying Jim and Arte protecting a member of an investment group whose last surviving member would inherit the group’s assets).

Having an incentive to kill someone to earn a profit is an example of what actuaries call a “moral hazard.” See Moral Hazard, INVESTOPEDIA, http://www.investopedia.com/terms/m/moralhazard.asp (last visited Jan. 16, 2015), archived at http://perma.cc/9DHX-FXK8 (defining “moral hazard” as “[t]he risk that a party to a transaction has not entered into the contract in good
Of course, tontines can be designed to avoid such mischief. For example, instead of distributing all of the contributions to the last survivor, a tontine could make periodic distributions. Historically, for example, governments issued tontines instead of regular bonds. In those tontines, the government would keep the tontine investors’ contributions but make high annual dividend payments to the tontine, dividing those payments among the surviving investors. When the last survivor died, the government had no further debt obligation. For example, in 1693, the English government issued a tontine to raise one million British pounds to help pay for its war against France. At a time when the regular bond interest rate was capped at 6%, King William's 1693 tontine, as it is known, entitled the surviving investors to share in 10% dividend payments to the tontine for the first 7 years and to 7% dividend payments thereafter.

Over the years, tontines like King William's became quite popular. At one point, Alexander Hamilton, the United States's first Secretary of the Treasury, suggested that the United States could use a tontine to pay off its Revolutionary War debt. All in all, government tontines played an important role in government finances over a couple of centuries, but they have since disappeared.

After the Civil War, tontines emerged as a popular investment for individuals in the United States, but they fell out of favor at the beginning of the twentieth century. The problem was not with the tontine form but faith, has provided misleading information about its assets, liabilities or credit capacity, or has an incentive to take unusual risks in a desperate attempt to earn a profit before the contract settles).
with embezzlement and fraud by the holders of tontine funds.\textsuperscript{13} Investigations of the insurance industry in New York led to the enactment of legislation in 1906 that all but banned tontines, and tontines have since been replaced by life insurance and similar financial products.\textsuperscript{14}

We believe that the time has come to revive tontines as a way of providing reliable, pension-like income for retirees. Specifically, we believe that variations on the tontine principle—that the share of each member of the tontine, at her death, is enjoyed by the survivors—can be used to develop a variety of attractive retirement-income financial products. For example, tontines could be used to create “tontine annuities” that could be sold to individual investors.\textsuperscript{15} These tontine annuities would make periodic distributions to surviving investors, but unlike traditional tontines, tontine annuities would solicit new investors to replace those that have died.\textsuperscript{16} Structured in this way, a tontine annuity could operate in perpetuity.\textsuperscript{17}

In this Article, we consider how the tontine principle could be used to create “tontine pensions” through which large employers could provide retirement income for their employees. These tontine pensions would have several major advantages over most of today’s pensions, annuities, and other retirement income products.

At the outset, Part I of this Article explains how the current U.S. retirement system works and how retirees can use pensions, annuities, and other financial products to generate retirement income.

Next, Part II offers a step-by-step explanation of how tontine funds, tontine annuities, and tontine pensions could work today. It then compares tontine pensions with traditional defined benefit pension plans, defined

\textsuperscript{13} See McKeever, supra note 9, at 511 (“The contemporary assessment . . . is that the tontine aspect of the standard insurance policies served as a distraction and scapegoat in coming up with remedies for the range of vices in the industry. The problem was not with the form, but with self-dealing management.” (footnote omitted)).

\textsuperscript{14} See COOPER, supra note 9, at 43-57 (discussing the findings of the Armstrong Committee, a committee created by the New York legislature to investigate the life insurance business, which led to legislation virtually banning tontine policies by forbidding insurance companies from deferring dividend payments beyond one year); see also Tom Baker & Peter Siegelman, Tontines for the Young Invincibles, REGULATION, Winter 2009–2010, at 26, available at http://object.cato.org/sites/cato.org/files/serials/files/regulation/2009/11/v32n4-4.pdf (describing anti-tontine regulations in New York and their effect on life and health insurance companies).


\textsuperscript{16} Id. at 12, 22.

\textsuperscript{17} Id. at 22.
contribution plans, and so-called “hybrid pensions” (e.g., cash balance plans). In particular, Part II shows that tontine pensions would have two major advantages over traditional pensions. First, unlike traditional pensions—which are frequently underfunded—tontine pensions would always be fully funded. Second, unlike a traditional pension—in which the pension plan sponsor must bear all the investment and actuarial risks—with a tontine pension, the plan sponsor bears neither of those risks. These two features should make tontine pensions a particularly attractive alternative for employers who wish to provide retirement income security for their employees but want to avoid the risks associated with a traditional pension.

Part III then develops a model tontine pension for a typical large employer. We then use that model to estimate the benefits that would be paid to retirees. For simplicity, the model assumes that, each year, an employer would contribute 10% of each employee’s salary to a tontine pension (in the real world, employers could choose to contribute a greater or lesser percentage of salary on behalf of their employees). The model generates tontine pension benefits for each retiree that would closely resemble an actuarially fair variable annuity—i.e., one without high insurance company fees (“loads”). Specifically, unlike commercial annuities which must support insurance agent commissions, insurance company reserves, risk-taking, and profits, the management and recordkeeping fees associated with running a tontine pension would be minimal. That means that tontine pensions would provide significantly higher retirement benefits than commercial annuities.

Part IV shows how such a model tontine pension could be used to replace a typical, large, traditional pension plan like the California State Teachers’ Retirement System (CalSTRS). Like so many other state-run pension plans, CalSTRS is underfunded; for example, as of June 30, 2013, CalSTRS was just 66.9% funded, with an unfunded liability of almost $74 billion. While replacing CalSTRS with a tontine pension would do nothing to reduce that $74 billion obligation, it would ensure that California

---


would never again have to worry about underfunding attributable to future benefit accruals.

Finally, Part V discusses how to solve some of the technical problems that would arise in implementing a tontine pension.

I. PENSIONS, ANNUITIES, AND OTHER LIFETIME INCOME MECHANISMS TODAY

Longevity risk—the risk of outliving one’s retirement savings—is probably the greatest risk facing current and future retirees. At present, for example, a 65-year-old man has a 50% chance of living to age 88 and a 25% chance of living to age 96, and a 65-year-old woman has a 50% chance of living to age 90 and a 25% chance of living to age 97. The joint life expectancy of a 65-year-old couple is even more remarkable: there is a 50% chance that at least one 65-year-old spouse will live to age 94 and a 25% chance that at least one will live to 100. In short, most individuals and couples will need to plan for the possibility of retirements that can last for 30 years or more.

Elderly Americans can generally count on Social Security benefits to cover at least a portion of their retirement income needs. In addition, retirees use pensions, annuities, and a variety of other mechanisms to ensure that they have adequate incomes throughout their retirement years. These financial mechanisms are discussed in turn.

A. Social Security

Social Security provides monthly cash benefits to most retirees and their families. A worker builds Social Security protection by working in

---


22 Id.

employment that is covered by Social Security and paying the applicable payroll taxes. 24 Workers over age 62 generally are entitled to Social Security retirement benefits if they have worked in covered employment for at least 10 years. 25 Benefits are based on a measure of the worker’s earnings history in covered employment. Most importantly, benefits are indexed each year for inflation as measured by the Consumer Price Index. 26 While historically “full retirement age” was age 65, it is currently age 66, and it is gradually increasing to age 67 for workers born after 1959 (who will reach age 67 in or after 2027). 27 In June 2014, Social Security paid retirement benefits to 38.5 million retired workers, and the average monthly benefit paid to a retired worker was $1300.04. 28


25 See 42 U.S.C. § 402(a) (2012) (describing eligibility for old-age insurance benefits); id. § 414(a)(2) (defining a “fully insured individual” as, among other definitions, an individual having at least “40 quarters of coverage”).


The United States has a voluntary pension system, and employers can decide whether and how to provide pension benefits to their employees. However, when employers do provide pensions, those pensions are typically subject to regulation under the Employee Retirement Income Security Act of 1974 (ERISA).

1. Retirement Savings Are Tax-Favored

Most pension plans qualify for favorable tax treatment. Basically, employer contributions to a pension are not taxable to the employee; the pension fund’s earnings on those contributions are tax-exempt; and workers pay taxes only when they receive distributions of their pension benefits. Nevertheless, the employer is allowed a current deduction for its


32 Id. § 501(a).

33 Id. §§ 72(a)(1), 402(b)(2). See generally IRS, PENSION AND ANNUITY INCOME (2015), available at http://www.irs.gov/pub/irs-pdf/p575.pdf (explaining the tax treatment of distributions from pension and annuity plans). In general, a participant’s pension benefits will be fully taxable if the participant’s employer contributed all of the costs for the pension without including any of the contributions in the employee’s taxable wages. Id. at 11. On the other hand, if an individual made after-tax contributions to a pension or annuity, she can exclude part of her pension or annuity distributions from income. Id. More specifically, under I.R.C. §§ 72 and 402, the individual can exclude a fraction of each benefit payment from income. That fraction (the “exclusion ratio”) is based on the amount of premiums or other after-tax contributions made by the individual. I.R.C. §§ 72(b), 402(c) (2012); see also IRS, supra, at 11-15 (explaining the calculation of the amount of pension payments that can be excluded from income). The exclusion ratio enables the individual to recover her own after-tax contributions tax free and to pay tax only on the remaining portion of benefits which represents income. IRS, supra, at 11-15. Taxpayers who began receiving annuity payments from a qualified retirement plan after November 18, 1996 generally can use the so-called “Simplified Method” to calculate the tax-free part of their benefits. Id. at 12-13. Under the Simplified Method, the Code provides a table with a fixed number of anticipated payments that depends upon the annuitant’s age as of the annuity starting date. Id. The taxpayer then divides her total after-tax contributions over the applicable number of anticipated payments and excludes the amount so determined each year. Id.
contributions, within limits. Favorable tax rules are also available for individual retirement accounts (IRAs) and Roth IRAs.

2. Types of Pension Plans

Pension plans generally fall into two broad categories based on the nature of the benefits provided: defined benefit plans and defined contribution plans.

a. Defined Benefit Plans

In a defined benefit plan, an employer promises its employees a specific benefit at retirement. To provide that benefit, the employer typically makes payments to a trust fund, the fund grows with investment returns, and eventually the employer withdraws money from the trust fund to pay the promised benefits. Employer contributions are based on actuarial valuations, and the employer bears all of the investment risks and responsibilities.

For example, a plan might provide that a worker’s annual retirement benefit \( B \) is equal to 2% multiplied by the number of years of service \( yos \) multiplied by final average compensation \( fac \) \( B = 2\% \times yos \times fac \). Under this traditional, final-average-pay formula, a worker who retires after 30 years of service with a final average compensation of $50,000 would receive a pension of $30,000 a year for life \( ($30,000 = 2\% \times 30\ yos \times $50,000 \ fac) \). While many defined benefit plans allow for lump-sum distributions, the

---

34 I.R.C. § 404(a) (2012).
35 Id. § 219(a). Almost any worker can set up an IRA with a bank or other financial institution. In 2015, individuals without pension plans can contribute and deduct up to $5500 to an IRA, although individuals over age 50 can contribute and deduct another $1000 (for a total of up to $6500), and spouses can contribute and deduct similar amounts. Press Release, IRS, IRS Announces 2015 Pension Plan Limitations; Taxpayers May Contribute up to $18,000 to their 401(k) Plans in 2015 (Oct. 23, 2014), available at http://www.irs.gov/uac/Newsroom/IRS-Announces-2015-Pension-Plan-Limitations-1.
36 I.R.C. § 408A (2012). Unlike regular IRAs, contributions to Roth IRAs are not tax deductible. Id. § 408A(c)(1). Instead, withdrawals are tax free. Id. § 408A(d)(1). Like regular IRAs, however, the earnings on Roth IRA investments are tax exempt. Id. § 408A(d)(2).
37 FORMAN, supra note 23, at 215.
38 Id.
39 Id.
40 Final average compensation is often computed by averaging the worker’s salary over the last three or five years prior to retirement. Alternatively, some plans use career average compensation instead of final average compensation. Under a career earnings formula, benefits are based on a percentage of an average of an employee’s career earnings for every year of service by the employee. Id.
default benefit is a retirement income stream in the form of an annuity for life. 41

Traditional defined benefit plans in the real world are often underfunded for a variety of reasons. 42 For example, the Pension Benefit Guaranty Corporation (PBGC) has already bailed out thousands of failed private-sector pension plans. Indeed, according to its 2013 annual report, the PBGC paid $5.5 billion to 900,000 retirees in more than 4600 failed private pension plans in its 2013 fiscal year, and the PBGC expects that another 620,000 workers will receive benefits when they retire. 43 Likewise, at the end of July 2014, pension plans sponsored by S&P 1500 companies were only 85% funded, reflecting a collective deficit of $340 billion. 44 Many government pension plans are also underfunded. On average, state public pensions in the United States were only 70.9% funded in 2012, reflecting cumulative unfunded liabilities of $894 billion. 45

b. Defined Contribution Plans

Under a typical defined contribution plan, the employer simply determines a specified percentage of a worker’s compensation that should be set aside and then contributes that percentage to an individual investment account

---

41 In the United States, defined benefit plans are generally designed to provide annuities, i.e., “definitely determinable benefits . . . over a period of years, usually for life, after retirement.” Treas. Reg. § 1.401-1(b)(1)(i) (2012).

42 Traditional defined benefit plans can easily become underfunded for three reasons: (1) the employers promise their employees additional benefits for past service, (2) the employers fail to make their actuarially required contributions, or (3) the assets held in the plan decline in value because of market volatility.


for that worker. For example, contributions might be set at 10% of annual compensation. Under such a plan, a worker who earned $50,000 in a given year would have $5000 contributed to an individual investment account on her behalf ($5000 = 10% × $50,000). Her benefit at retirement would be based on all such contributions, plus investment earnings. Unlike traditional defined benefit plans, defined contribution plans usually make distributions in lump sum or periodic distributions rather than life annuities.

There are many different types of defined contribution plans, including money purchase pension plans, savings and thrift plans, deferred profit-sharing plans, savings incentive match plans (SIMPLE), simplified employee pensions (SEPs), and employee stock ownership plans (ESOPs). Most notably, according to Internal Revenue Code section 401(k), profit-sharing and stock bonus plans often include a feature that allows workers to choose between receiving cash currently or deferring taxation by placing the money in a trust. Consequently, these plans are often called "401(k) plans," and they are the most popular type of retirement plan in the United States. The maximum amount of such elective deferrals that can be made by an individual in 2015 is $18,000, although workers over the age of 50 can contribute another $6000 (for a total of up to $24,000). Since 2006, employers have also been permitted to set up Roth 401(k) plans.


47 Defined contribution plans are also known as "individual account" plans because each worker has her own account, as opposed to defined benefit plans, in which the plan’s assets are pooled for the benefit of all of the employees.


51 BLS Examines Popular 401(k) Retirement Plans, PROGRAM PERSP. (U.S. Bureau of Labor Statistics, Washington, D.C.), Nov. 2010, at 1, available at http://www.bls.gov/opub/perspectives/program_perspectives_vol2_issue6.pdf (asserting that there has been a “wide-spread movement towards defined contribution plans, such as 401(k) and 403(b) . . . in private industry and to a lesser extent, in State and local government”).

52 I.R.C. § 402A(b)(i) (2012). (“The term ‘qualified Roth contribution program’ means a program under which an employee may elect to make designated Roth contributions in lieu of all or a portion of elective deferrals the employee is otherwise eligible to make under the applicable
Because retirement benefits are based on the retiree’s individual account balance, benefits can vary dramatically depending upon investment returns and interest rates. For example, over the past decade, a withdrawal strategy based on taking 4% of the balance in a retiree’s account annually would have led to dramatically different payouts in the peak stock market years of 2007 and 2014, as opposed to the bottom of the recession in 2009. Using an account balance to buy an annuity would also not fully offset those risks, as fixed annuity payouts vary with market interest rates, and variable annuity payouts vary with the performance of the underlying assets (just as they would with payouts under a 4% strategy).

c. Hybrid Retirement Plans

So-called “hybrid” retirement plans mix the features of defined benefit and defined contribution plans. For example, a cash balance plan is a defined benefit plan that closely resembles a defined contribution plan. Like other defined benefit plans, employer contributions are based on actuarial valuations, and the employer bears all of the investment risks and responsibilities. Like defined contribution plans, however, cash balance plans provide workers with individual accounts (albeit hypothetical). A simple cash balance plan might allocate 10% of salary to each worker’s account annually and credit the account with 5% interest on the account’s balance. Under such a plan, a worker who earns $50,000 in a given year

54 The Dow Jones Industrial Average hit 14,000 in October 2007, fell to around 7,000 in February 2009, and rose to more than 17,000 in September 2014. Dow Jones Industrial Average, GOOGLE FINANCE, https://www.google.com/finance?q=INDEXDJX%3A.DJI&ei=bXBqUsidGJCIxQriticaladid3 (last visited Jan. 16, 2015) (follow “Historical Prices” hyperlink, set daily price time period, then follow “update” hyperlink); see also infra subsection I.C.1 for a discussion of the so-called 4% rule.

55 See The Dangers of Buying an Annuity When Interest Rates are Low, ANNUITY DIG., http://www.annuitydigest.com/blog/tom/dangers-buying-annuity-when-interest-rates-are-low (last visited Jan. 16, 2015), archived at http://perma.cc/R8Y9-TKJM (warning how interest rate fluctuations can cause annuities to become very expensive because fixed annuity payments are based on prevailing interest rates).


58 Id.
would receive an annual cash balance credit of $5000 ($5000 = 10% × $50,000), plus an interest credit equal to 5% of the balance in her hypothetical account as of the beginning of the year.

3. The Regulation of Employment-Based Plans

Since ERISA's enactment, an entire system has emerged to regulate pensions. ERISA requires significant reporting and disclosure in the administration and operation of employee benefit plans. ERISA also imposes extensive fiduciary responsibilities on employers and administrators of employee benefit plans. ERISA and the Internal Revenue Code also impose many other requirements on retirement plans, including rules governing normal retirement

---


60 I.R.C. § 401(a) (2012) ("A trust created or organized in the United States and forming part of a stock bonus, pension, or profit-sharing plan of an employer for the exclusive benefit of his employees or their beneficiaries shall constitute a qualified trust under this section . . . ."); Employee Retirement Income Security Act of 1974 § 403, 29 U.S.C. § 1103(a) (2012) ("Except as provided in subsection (b) of this section, all assets of an employee benefit plan shall be held in trust by one or more trustees.").


age, participation, coverage, vesting standards, benefit accrual, limitations on contributions and benefits, nondiscrimination, and minimum funding standards.

Pertinent here, federal laws outside of ERISA and the Internal Revenue Code can also impose limits on pension plans. For example, even though women tend to have longer life expectancies than men, Title VII of the Civil Rights Act of 1964 bars pension plans from requiring higher contributions from women than men or paying women lower benefits than men.

C. Other Sources of Lifetime Income

In addition to accumulating retirement assets through the Social Security and pension systems, individuals can save on their own. Investment income is generally subject to federal personal income tax rates of up to 39.6% in 2015; however, dividend income and capital gains are generally taxed at no more than a 20% rate. Also, there are various tax advantages associated

---

65 I.R.C. § 410(b) (2012).
69 Id. § 401(a)(4).
71 See supra text accompanying note 21.
with investing in homes,75 state and local bonds,76 annuities,77 and life insurance.78

Retirees can use a variety of approaches to generate retirement income from their voluntary savings.79 One approach is for retirees to commit to systematic withdrawals of, for example, 4% of their account balances each year—a strategy that has a relatively low risk of ruin (running out of money before death).80 Traditional lifetime annuities offer another approach for spreading retirement savings out over a lifetime. Another alternative involves buying longevity insurance, for example, buying a deferred annuity at age 65 that starts making payments only if the annuitant lives past age 85.81 Retirees can also invest in other financial products that can provide guaranteed lifetime benefits. These are discussed in turn.

1. Systematic Withdrawals

One of the simplest and most common strategies for managing retirement savings is to invest all of the retirement savings in a diversified portfolio and then use a conservative withdrawal rate and a systematic withdrawal plan (SWP) designed to have a high probability that the retirement savings will

75 For example, home mortgage interest is generally deductible, and gains from the sale of a personal residence are often excludable. Id. §§ 121, 163(a)-(b).
76 For example, gross income does not include interest on any state or local bond. Id. § 103.
77 See supra note 33 for a more in-depth explanation of how an annuitant can often exclude a fraction of each annuity payment from income under I.R.C. § 72 (2012).
81 Id. § 3.01.
last for 20 or 30 years. In that regard, financial planners often suggest following the so-called “4% rule.” The basic idea is to set spending at 4% of retirement savings and invest those savings in a portfolio with 50% stocks and 50% bonds. Each year thereafter, spending is increased to keep up with inflation. For example, assuming that an individual has a $1,000,000 nest egg, in the first year of retirement she would withdraw 4% ($40,000), and each year thereafter that dollar amount would increase to keep up with inflation. Assuming a 3% annual inflation rate, annual withdrawals would increase to $41,200 in the second year, $42,436 in the third year, and so on. While there is a possibility of running out of money before death, many financial planners believe this strategy can usually work for 30 years. To minimize the prospect of outliving one’s nest egg in the recent economic recession, however, some financial advisors advised retirees to skip their scheduled inflation adjustments or to withdraw less than 4% of their new balances.

---

82 Id. § 3.03[4].
83 See William P. Bengen, Determining Withdrawal Rates Using Historical Data, J. FIN. PLAN., Oct. 1994, at 174-75 (explaining, using historical data, why retirees should withdraw no more than 4% of their retirement savings each year); see also JANEMARIE MULVEY & PATRICK PURCELL, CONG. RESEARCH SERV., R40008, CONVERTING RETIREMENT SAVINGS INTO INCOME: ANNUITIES AND PERIODIC WITHDRAWALS 17 (2008) (“A large body of research on safe withdrawal rates for individuals has determined that a real withdrawal rate in the neighborhood of 4 percent of the initial portfolio has a low chance of running out of money.” (internal quotation marks omitted)); Benjamin Bridges, Robert Gesumaria & Michael V. Leonesio, Assessing the Performance of Life-Cycle Portfolio Allocation Strategies for Retirement Saving: A Simulation Study, SOC. SECURITY BULL., 2010, at 23 (examining the performance of life-cycle portfolio allocation strategies with varying exposure to stock and bond market risk based on observed historical U.S. asset returns).
84 Bengen, supra note 83, at 175.
2. Lifetime Annuities

Traditional lifetime annuities can also provide lifetime retirement income. For example, for a 65-year-old man who purchased a $100,000 immediate, level-payment annuity without inflation protection as of January 1, 2014, the annual payout would be around $6864 or 6.86% of the annuity’s purchase price. Because women tend to live longer than men, the annual payout for a 65-year-old woman who elected an immediate, level-payment annuity as of January 1, 2014 would be only $6408, or 6.41% of the annuity’s purchase price.

With inflation-adjusted annuities, annual payouts would start lower but could end up higher. For example, if the hypothetical 65-year-old man instead chose an annuity stream with a 3% escalator, the annual payout for the first year would be just $5064.

3. Longevity Insurance

Alternatively, retirees can protect against longevity risk by purchasing longevity insurance. The typical approach is to buy a “deferred annuity” at age 65 that starts making annual payments only if the annuitant lives past age 80 or 85. For example, in February 2012, a 65-year-old man could have invested $100,000 in a MetLife deferred annuity, and beginning at age 85,...
he would receive a level lifetime income of $25,451.04 per year.\footnote{E-mail from Hersh Stern, WebAnnuities Ins. Agency, Inc., to Jonathan Barry Forman (Feb. 7, 2012, 11:46 EST) (on file with authors). Alternatively, that 65-year-old man could have purchased a deferred annuity that starts at age 80 and pays $17,069.40 per year; at age 75 and pays $11,649.84 per year; or at age 70 and pays $8,133.60 per year. Id. Companies do not offer inflation-adjusted deferred annuities, but some companies do offer fixed step-ups. Joseph A. Tomlinson, Income Choices, FIN. PLAN. (May 1, 2011), http://www.financial-planning.com/fp_issues/2011_5/income-choices-2672801-1.html, archived at http://perma.cc/U35E-EXKR (comparing various investment strategies including systematic withdrawals, immediate annuities, deferred annuities, and guaranteed lifetime withdrawal benefits).}

Therefore, with a relatively small upfront investment, a retiree can secure an income stream that starts sometime in the future. The retiree can then use the rest of her savings to cover the fixed number of years until the year that the deferred annuity payments start.\footnote{See, e.g., Stephen C. Sexauer, Michael W. Peskin & Daniel Cassidy, Making Retirement Income Last a Lifetime, FIN. ANALYSTS J., Jan.–Feb. 2012, at 76-77 (proposing a “decumulation benchmark” that would use about 88% of retiree savings to purchase a laddered portfolio of Treasury Inflation-Protected Securities [TIPS] for the first 20 years and a deferred life annuity purchased with the remaining 12%); Rick Wurster, DC 20/20: Pathways to a Secure Retirement, ROTMAN INT’L J. PENSION MGMT., Fall 2011, at 54, 58 (suggesting that an annuity providing 35% of real income replacement from age 85 would cost about 7.5% of a participant’s average account balance at retirement).} There is some risk of running out of money before the year that the deferred annuity starts, but that risk is certainly more manageable than trying to manage one’s retirement savings over the indefinite future.\footnote{Finally, it is worth noting that workers might be able to buy deferred annuities in installments, starting at a young age. For example, a worker could use a portion of her retirement savings each year to purchase a deferred annuity that starts at age 65, or at the advanced ages of 70, 75, 80, 85, or even 90. Accordingly, this type of deferred annuity product could be used to provide retirement benefits that mimic the lifetime pensions provided by traditional defined benefit plans. See Moshe A. Milevsky, Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities (ALDA), N. AM. ACTUARIAL J., Oct. 2005, at 109, 111 (“[T]he [Advanced-Life Delayed Annuity] is preferable to a pure endowment policy that would (mature and) pay a lump sum at age 80, 85, or 90 since it would continue to provide periodic lifetime income regardless of how long the annuitant lived beyond the endowed age.”); see also Zorast Wadia, Longevity Risk & Retirement, ACTUARIAL DIG., Spring 2012, at 4, available at http://publications.milliman.com/publications/eb-published/pdfs/longevity-risk-and-retirement.pdf (proposing a new retirement paradigm combining aspects of a defined benefit plan and a defined contribution plan).} 4. Other Lifetime Income Products

Retirees can also choose to purchase variable annuities with guaranteed lifetime withdrawal benefit (GLWB) funds to manage their longevity risk.\footnote{See Moshe A. Milevsky & Ling-wu Shao, Annuities and Their Derivatives: The Recent Canadian Experience (“[GLWB funds] provide savers with (some of) the retirement longevity protection of a traditional annuity, without forcing them to surrender upside potential or liquidity.”), in SECURING LIFELONG RETIREMENT INCOME: GLOBAL ANNUITY MARKETS AND POLICY 50, 56 (Olivia S. Mitchell, John Piggott & Noriyuki Takayama eds., 2011).}
A GLWB is based on a variable annuity, but it allows investors to lock in a minimum guarantee for life.\(^{96}\) Similarly, so-called “standalone living benefits” are like GLWBs, except that instead of using a variable annuity chassis, standalone living benefits use mutual funds or managed accounts as the base.\(^{97}\)

II. TONTINE PENSIONS

After analyzing the tontine principle, this Part discusses how to design a tontine fund, a tontine annuity, and finally, a tontine pension.

A. The Tontine Principle

In a simple tontine, members contribute equally to buy a portfolio of investments that is awarded entirely to the last surviving member.\(^{98}\) Alternatively, each time a member of a tontine pool dies, her account balance could be divided among the surviving members of the pool.\(^{99}\) The latter type of tontine could be used to develop new financial products that would provide reliable, pension-like income for retirees. The key point is that variations on the tontine principle—that the share of each, at death, is enjoyed by the survivors—can be used to create a variety of attractive retirement income financial products.\(^{100}\)

At the outset, imagine that 1000 65-year-old retirees each contribute $1000 to an investment fund that purchases a $1,000,000 Treasury bond paying 4% interest coupons.\(^{101}\) The bond will generate $40,000 in interest...
per year, which will be split equally among the surviving participants. A custodian holds the bond and, because the custodian takes no risk and requires no capital, the custodian charges a trivial fee. Assuming that all of the investors live through the first year, they will each receive a $40 dividend from the fund ($40 = $40,000/1000). If only 800 of the original investors are alive a decade after the tontine started (when the survivors are 75), then each will receive a $50 dividend ($50 = $40,000/800). If only 100 investors are alive two decades after that (when the survivors are 95), then each will receive a $400 dividend ($400 = $40,000/100). Later, when only 40 investors remain, each will receive a $1000 dividend ($1000 = $40,000/40). If the terms of the tontine call for liquidation at that point, then each of the 40 survivors would also receive a liquidating distribution of $25,000 ($25,000 = $1,000,000/40). Alternatively, the tontine could be designed so that the last survivor receives the entire $1,000,000.

Most retirees would likely prefer to have reasonably level benefits throughout their retirement years, rather than benefits that increase sharply at the very end of their lives. Accordingly, it would make sense to design tontine financial products with benefits that are level throughout retirement (like an immediate, level-payment annuity) or, alternatively, that increase gradually throughout retirement (like an immediate, inflation-adjusted annuity). Of particular note, unlike these commercial annuities—which must support insurance agent commissions and insurance company reserves, risk-taking, and profits—an early death in a tontine benefits only other investors, not some opportunistic insurance company. This limitation of benefits to investors should make tontines very popular.102

B. A Tontine Fund

Before explaining how the tontine principle can be used to create a tontine pension, this Section shows how the tontine principle can be used to

102 For example, Professor Suzanne Shu suggests that a tontine for one's fellow firefighters will be perceived as fairer than the typical commercial annuity that they could buy from an insurance company: with a commercial annuity, an early death seems to benefit the insurance company, but with a tontine, an early death benefits fellow firefighters. SHLOMO BENARTZI, BEHAVIORAL FINANCE AND THE POST-RETIREMENT CRISIS: A RESPONSE TO THE DEPARTMENT OF THE TREASURY/DEPARTMENT OF LABOR REQUEST FOR INFORMATION REGARDING LIFETIME INCOME OPTIONS FOR PARTICIPANTS AND BENEFICIARIES IN RETIREMENT PLANS 15 (2010), available at http://www.dol.gov/ebsa/pdf/1210-AB33-617.pdf.
create a tontine fund. The next Section explains how to create a tontine annuity.\footnote{For background on tontine annuities, see, for example, Goldsticker, \textit{supra} note 100; Milevsky & Salisbury, \textit{supra} note 1; Michael J. Sabin, \textit{A Fast Bipartite Algorithm for Fair Tontines} (May 22, 2011) (unpublished manuscript), available at http://ssrn.com/abstract=1848737; Sabin, \textit{supra} note 15.}

We have already shown how a tontine fund could work for a group of 65-year-old investors who all invested the same amount (i.e., $1000)\footnote{See \textit{supra} text accompanying note 101.}. This Section shows how to create a tontine fund that is fair to all investors, regardless of their age, gender, or the amount of their investments.

In a simple tontine, when a member dies, the balance in her account (i.e., her contribution plus investment earnings) is distributed to the surviving members of the pool as “mortality gains.”\footnote{Individuals who invest in annuity-like products have mortality gains and losses depending on when they die. Individuals who live longer than their peers get mortality gains from those who precede them, while individuals who die earlier than their peers suffer mortality losses. See David Blake, \textit{Annuity Markets: Problems and Solutions}, 24 GENEVA PAPERS ON RISK & INS. 358, 371 (1999) (explaining that a mortality cross-subsidy “arises because some annuitants will die shortly after taking out an annuity thereby releasing a ‘mortality profit’ which insurance companies share with longer-surviving annuitants”).} Those forfeitures are divided equally among the survivors. Unfortunately, that approach results in an unfair situation because it favors younger members who are likely to live longer and receive more distributions.

In a tontine fund with participants who have different ages, genders, and investment levels, the surviving members should not get equal portions of a dying member’s balance. Instead, the distributions should be made in unequal portions, carefully chosen to provide fair bets for all investors. In short, a tontine fund should be governed by a “fair transfer–plan” that accounts for each member’s life expectancy (i.e., death probability) and investment level.\footnote{The term “fair transfer–plan” is derived from Sabin, \textit{supra} note 15, at 5.} In this Section, we describe how such a tontine fund would be designed.

1. A Fair Transfer–plan

We can design a fair transfer–plan (FTP) to build a tontine fund that provides fair bets for all investors. The concept is straightforward: members join the tontine fund by contributing a desired amount, and each time a member dies, her contribution (and investment earnings) is distributed to the surviving members according to the FTP. New members may join at any time by making a contribution of a desired amount; however, no
member may ever withdraw her contributions (or investment earnings).\textsuperscript{107} Structured in this way, a tontine fund could operate into perpetuity.

a. \textit{Tontine Funds Can Be Fair to Members of Different Ages}

Tontine funds can easily be designed to be fair to members of different ages. For example, Table 1 illustrates a tontine fund with just four members of different ages. To keep this example as simple as possible, we assume that each member \((i)\) has contributed $1000 to the fund and that these contributions do not earn any interest.\textsuperscript{108} We use unisex life tables rather than gender-based life tables.\textsuperscript{109} For example, member 4 in Table 1 is an 80-year-old who has a life expectancy \((e_i)\) of 8.95 years, and a 5.2\% chance of dying before reaching age 81 (i.e., a death probability \((q_i)\) of 0.051906).

Table 1: A Tontine Fund with Four Members of Different Ages, Unisex\textsuperscript{110}

<table>
<thead>
<tr>
<th>Member ((i))</th>
<th>Age ((x_i))</th>
<th>Life Expectancy ((e_i)) (years)</th>
<th>Death Probability ((q_i))</th>
<th>Force-of-Mortality Probability ((f_i))</th>
<th>Fair Transfer-plan Weight ((w_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>18.88</td>
<td>0.013181</td>
<td>0.013269</td>
<td>0.053815</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>15.22</td>
<td>0.020314</td>
<td>0.020523</td>
<td>0.086183</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>11.89</td>
<td>0.032111</td>
<td>0.032638</td>
<td>0.146795</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>8.95</td>
<td>0.051906</td>
<td>0.053302</td>
<td>0.73207</td>
</tr>
</tbody>
</table>

Table 1 also shows a parameter known as the force-of-mortality probability \((f_i)\). Here is the logic: suppose that at time \(t\) a member of the pool dies. Pretend that we do not know which member has died at time \(t\). The force-of-mortality probabilities indicate the relative probability of death for each member of the pool. If, at the instant that a member died, one member has a force-of-mortality probability with a value \(f_i\), and another has a value \(2f_i\), then the second member is twice as likely as the first to be the one who.

\textsuperscript{107} The situation is identical to a commercial annuity: once the premium is paid, there is no refund.

\textsuperscript{108} That is, the underlying investments do not pay interest or dividends, nor are there any sales that result in gains or losses. We relax this assumption later in the paper. See infra subsection II.B.1.d.

\textsuperscript{109} The life expectancies \((e_i)\) and death probabilities \((q_i)\) in Table 1 are derived from data provided to the authors by the Social Security Administration. E-mail from K. Mark Bye, Soc. Sec. Admin., to Jonathan Barry Forman (Nov. 12, 2013, 14:31 EST) (on file with authors). See infra Appendix Table 1 for a fuller version of the Social Security Administration 2009 unisex life table.

\textsuperscript{110} Table 1 is drawn from Bye, supra note 109, and the authors’ computations.
died. In Table 1, for example, member 4 (our 80-year-old) has a relatively large force-of-mortality probability (0.053302), while member 1 (our 65-year-old) has a relatively small force-of-mortality probability (0.013269). In short, member 4 is clearly the member who is the more likely of the two to have died at time \( t \). Indeed, of the four members in Table 1, member 4 is the most likely to die next. These force-of-mortality probabilities \( (f_i) \) are relatively easy to compute from the death probabilities \( (q_i) \) in a mortality table.\footnote{111}

Table 1 also shows another parameter, referred to as the “fair transfer–plan weight” \( (w_i) \). When a member of a tontine fund dies, she forfeits her entire contribution. Her contribution is then divided among the surviving members, with each surviving member receiving some fraction of the decedent’s account. For example, if member 4 (the 80-year-old) is, in fact, the member who died next, her $1000 contribution would be distributed to members 1, 2, and 3 based on their respective fair transfer–plan weights \( (w_i) \). These fair transfer–plan weights \( (w_i) \) are relatively easy to compute from the force-of-mortality probabilities \( (f_i) \).\footnote{112}

111 The force-of-mortality probabilities in Table 1 were computed from the death probabilities \( (q_i) \) in Column 4 of that table. See Sabin, supra note 15, at 10-12 (demonstrating how the force-of-mortality method is interpolated from the probability of death during a given year).

The explanation is as follows: at the outset, we make the simplifying assumption that the force of mortality is constant during each year of age. Next, suppose that the probability of dying during a specific year of age is 5%. Then, the probability of surviving the year is 1 - 0.05 = 95%. Now suppose the probability of surviving the first 6 months is 1 - 0.05/2 = 97.5%, and the probability of surviving the second 6 months is the same. Then, the probability of surviving the year is \( (0.975)^2 = 95.0625\% \). Now suppose the probability of surviving the first month is 1 - 0.05/12, and the same for the second, third month, etc. Then, the probability of surviving the year is \( (1 - 0.05/12)^{12} = 95.123\% \). Generalizing this math, if the probability of surviving each of \( n \) periods within the year is 1 - 0.05/\( n \), then the probability of surviving the year is \( (1 - 0.05/n)^n \). As \( n \) grows to infinity, the probability of surviving the year becomes \( e^{-0.05} = 95.123\% \), where \( e \) is Euler’s number (~2.71828). The probability of dying sometime during the year (i.e., the death probability) is 1 - 0.95123 = 4.877%, and the force-of-mortality probability is 5%.

Now, let us work it in reverse. Suppose the mortality table says that the death probability during a specific year is 5%. What is the force-of-mortality probability for the year? It is the value \( x \) that satisfies \( e^x = 1 - 0.05 \). The solution is \( x = -\ln(1 - .05) = 5.129\% \), where “\( \ln \)” is the natural logarithm.

Accordingly, the force-of-mortality probabilities in Table 1 were computed from the death probabilities in Table 1 by using the formula, \( f_i = -\ln(1 - q_i) \). For example, for member 4, \( f_4 = -\ln(1 - 0.051906) = -\ln(1 - 0.05/12) = 0.053302 \). Of note, the force-of-mortality probabilities are fairly close in value to the death probabilities, except at older ages. See infra Appendix Table 1 (showing how the values in Columns 3 and 4 diverge as individuals live beyond age 100).

112 The explanation is as follows: our goal is to design a fair transfer–plan, one that provides fair bets to all of the members. This means we want the expected return \( (ER_i) \) received by each member \( i \) to be zero. Mathematically, we want

\[
0 = -f_i s_i - \sum_{j \neq i} f_j s_j w_i / (1 - w_j)
\]

for each member \( i \).
More specifically, if member \( j \) dies, each surviving member \( i \) would receive some fraction of \( j \)'s $1000 contribution: the fraction that each member \( i \) would receive of member \( j \)'s contribution (\( s_j \)) is equal to \( w_i/(1 - w_j) \), for \( i \neq j \). The fair transfer–plan weights (\( w_i \)) are positive values that sum to 1, so the denominator \((1 - w_j)\) is the sum of all fair transfer–plan weights (\( w_i \)) except that of member \( j \). Meanwhile, member \( j \) would forfeit her entire $1000 contribution.

Finally, we can use the fair transfer–plan weights to determine the amounts that each member \( i \) would receive when member \( j \) dies. For example, if member 4 (the 80-year-old) dies, then member 1 (the 65-year-old) would receive $187.64 = $1000 \times \frac{w_1}{(1 - w_4)} = $1000 \times \frac{0.053815}{(1 - 0.713207)}; member 2 (the 70-year-old) would receive $300.51 = $1000 \times \frac{w_2}{(1 - w_4)} = $1000 \times \frac{0.086183}{(1 - 0.713207)}; member 3 (the 75-year-old) would receive $511.85 = $1000 \times \frac{w_3}{(1 - w_4)} = $1000 \times \frac{0.146795}{(1 - 0.713207)}; and, of course, member 4 would forfeit her $1000.\footnote{Checking our answer, $187.64 + $300.51 + $511.85 = $1000.}

We call the distributions to members 1, 2, and 3 "mortality-gain distributions"; meanwhile, member 4 has a mortality loss.\footnote{See supra note 105 and accompanying text.}

where: \( f_i \) is the force-of-mortality probability of member \( i \), \( s_i \) is the contribution made by member \( i \), and \( w_i \) is the fair transfer–plan weight for member \( i \) that we need to provide fair bets. See Sabin, A Fast Bipartite Algorithm for Fair Tontines, supra note 103, at 7-8 (explaining the underlying algorithm).

The formula above gives us a set of \( m \) equations, one equation for each member \( i \). The solution to those equations is unique, meaning there is only one set of fair transfer–plan weights (\( w_i \)) that solve those equations. The challenging part is that the equations are not linear because, in each equation, one unknown, \( w_i \), is divided by another unknown, \((1 - w_j)\). That means we cannot solve the equations using the standard methods of linear algebra. Fortunately, however, we are able to solve these equations by using an iterative method designed specifically for them. More specifically, the iterative method uses a bisection algorithm. See id. at 12-13 (demonstrating the bisection algorithm method). While the explanation of how to create the computer program to solve that algorithm is too involved to explain here, we can easily show that the method works, as the fair transfer–plan weights (\( w_i \)) in Table 1 do solve the pertinent equations. For example, for \( i = 3 \), \( ER_3 = 0 \):

\[
\begin{align*}
&- 0.032638 \times $1000 = -32.638 \\
&+ 0.013269 \times $1000 \times 0.146795/(1 - 0.053815) = 2.059 \\
&+ 0.020523 \times $1000 \times 0.146795/(1 - 0.086183) = 3.297 \\
&+ 0.053302 \times $1000 \times 0.146795/(1 - 0.713207) = 27.283 \\
&= 0
\end{align*}
\]

We can verify that similar equations for \( i = 1, 2, \) and 4 also work. Therefore, we can be certain that the fair transfer–plan weights (\( w_i \)) in Table 1 accomplish our goal for a fair transfer–plan (i.e., \( ER_i = 0 \)).
In short, a tontine fund can fairly accommodate members of different ages. The key is to design a fair transfer-plan that uses each member’s death probability \(q_i\) to determine her force-of-mortality probability \(f_i\) and her fair transfer-plan weight \(w_i\). The result is a tontine investment fund that offers a fair bet to all members. It is worth noting that the iterative method used to determine the fair transfer-plan weights \(w_i\) is fast and could easily handle large tontine funds involving millions of members.

b. Tontine Funds Can Be Fair to Both Men and Women

Tontines can also be designed to take gender into account.\(^{115}\) Women tend to live longer than men and have lower death probabilities than same-aged men.\(^ {116}\) For example, Table 2 shows that the life expectancy \(e_i\) for a 65-year-old man in 2009 was 17.51 years, and his death probability \(q_i\) was 0.016182; meanwhile, the life expectancy \(e_i\) of a 65-year-old woman that year was 20.19 years and her death probability \(q_i\) was 0.010298.\(^ {117}\) Compare those numbers with their 18.88-year unisex life expectancy \(e_i\) and their 0.013181 unisex death probability \(q_i\) shown in Table 1.

<table>
<thead>
<tr>
<th>Member</th>
<th>Age</th>
<th>Gender</th>
<th>Life Expectancy ((e_i))</th>
<th>Death Probability ((q_i))</th>
<th>Force-of-Mortality Probability ((f_i))</th>
<th>Fair Transfer-plan Weight ((w_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>male</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.330931</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
<td>male</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.330931</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>female</td>
<td>20.19</td>
<td>0.010298</td>
<td>0.010351</td>
<td>0.169069</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>female</td>
<td>20.19</td>
<td>0.010298</td>
<td>0.010351</td>
<td>0.169069</td>
</tr>
</tbody>
</table>

A tontine fund can take gender into account by using gender-based death probabilities \(q_i\) rather than unisex death probabilities. For example, Table 2 illustrates a tontine fund with two men and two women. For simplicity, we assume that all the members of this tontine fund are age 65.

\(^{115}\) See, e.g., Sabin, supra note 15, at 14-16 (providing an example of a tontine that could be fair regardless of the participants’ gender).

\(^{116}\) See, e.g., supra text accompanying note 21.

\(^{117}\) The life expectancies \(e_i\) and death probabilities \(q_i\) in Table 2 are derived from data provided to the authors by the Social Security Administration. E-mail from K. Mark Bye, Soc. Sec. Admin., to Jonathan Barry Forman (Dec. 3, 2014, 10:03 EST) (on file with authors).

\(^{118}\) Table 2 is drawn from Bye, supra note 117, and the authors’ computations.
that each contributed $1000 to the fund, and that their contributions do not earn any interest. However, as the previous subsection showed, a tontine fund could easily accommodate members of different ages, as well.

Assuming that member 4 (a female) dies, then members 1 and 2 (the males) would each receive a mortality-gain distribution of $398.27 ($398.27 = $1000 \times \frac{w_1(1 - w_4)}{(1 - w_4)} = $1000 \times 0.330931/(1 - 0.169069)); member 3 (the other female) would receive a mortality-gain distribution of just $203.47 ($203.47 = $1000 \times \frac{w_3(1 - w_4)}{(1 - w_4)} = $1000 \times 0.169069/(1 - 0.169069)) and, of course, member 4 would forfeit her $1000 balance (a mortality loss of $1000). On the other hand, if these mortality-gain distributions had instead been determined under a unisex mortality table, it is easy to see that when one member dies each survivor would get one-third, $333.33 ($333.33 = $1000 \times \frac{w_i(1 - w_j)}{(1 - w_j)} = $1000 \times 0.25/(1 - 0.25))

Based on this comparison, the female members would appear to be short-changed if a tontine fund used a gender-based life expectancy table; however, remember that the 65-year-old females in any tontine fund are likely to live longer and receive more mortality-gain distributions than their 65-year-old brethren. All in all, the expected returns for both men and women would be equal, and both genders would get fair returns on their $1000 investments (i.e., fair bets).

Implicitly, since gender-based tontine funds would be fair to both women and men, unisex tontine funds must be “unfair” to one gender. In fact, unisex tontine funds would be unfair to men in precisely the same way that unisex commercial annuities are “unfair” to men: the annual distributions would be identical for men and women with a unisex tontine fund (or unisex annuity), but women tend to live longer and would likely collect more money from unisex tontine funds (and unisex annuities) than men.

The bottom line is that women would generally fare better than men in any tontine fund that used unisex life tables. Accordingly, to attract both male and female investors, the free market would force tontine funds to take gender into account in their design (i.e., use gender-based, not unisex, life tables), just as the free market today already forces insurance companies to take gender into account when they sell annuities.

In short, a tontine fund can fairly accommodate members of different genders by using gender-based life tables rather than unisex life tables.

---

119 Checking our answer, $398.27 + $398.27 + $203.47 = $1000.01 (error due to rounding).
120 That is, \(ER_i = 0\) for both women and men.
121 See supra notes 88-89 and accompanying text. We have much more to say about gender issues later in this Article. See infra Section V.D; see also Mary L. Heen, Nondiscrimination in Insurance: The Next Chapter, 49 GA. L. REV. (forthcoming 2015) (manuscript at 61) (on file with University of Pennsylvania Law Review) (arguing that gender discrimination laws should be expanded to prevent insurance companies from selling gender-based annuities).
c. Tontines Can Fairly Accommodate Members with Differing Levels of Contribution

Tontine funds can also allow members to make differing levels of contributions. For example, Table 3 illustrates a tontine fund with four members with different contribution levels ($s_i$). For simplicity, all of the members of this tontine fund are 65-year-old men (and contributions do not earn any interest), although as the previous subsections have shown, a tontine fund can easily accommodate members of different ages and genders.

Table 3: A Tontine Fund with Four Members, Different Levels of Contribution

<table>
<thead>
<tr>
<th>Member ($i$)</th>
<th>Age ($x_i$)</th>
<th>Contribution ($s_i$)</th>
<th>Life Expectancy ($e_i$)</th>
<th>Death Probability ($q_i$)</th>
<th>Force-of-Mortality Probability ($f_i$)</th>
<th>Fair Transfer-plan Weight ($w_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>$1000</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.066510</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
<td>$2000</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.145278</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>$3000</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.247530</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>$4000</td>
<td>17.51</td>
<td>0.016182</td>
<td>0.016314</td>
<td>0.540682</td>
</tr>
</tbody>
</table>

Mathematically, if the dying member is member $j$, then each surviving member $i$ would receive a mortality-gain distribution equal to $s_jw_i/(1 - w_j)$, for $i \neq j$. For example, assuming that member 4 (the $4000 contributor) dies, then member 1 (the $1000 contributor) would receive a mortality-gain distribution of $579.21 (579.21 = s_jw_i/(1 - w_j) = 4000 \times 0.06651/(1 - 0.540682)); member 2 (the $2,000 contributor) would receive a mortality-gain distribution of $1265.16 (1265.16 = s_jw_i/(1 - w_j) = 4000 \times 0.145278/(1 - 0.540682)); member 3 (the $3000 contributor) would receive a mortality-gain distribution of $2155.63 (2155.63 = s_jw_i/(1 - w_j) = 4000 \times 0.247530/(1 - 0.540682)); and, of course, member 4 would forfeit his $4000 balance (a mortality loss of $4000).\(^{123}\)

\(^{122}\) Table 3 is drawn from Bye, supra note 117, and authors’ computations.

\(^{123}\) Checking our answer, $579.21 + 1265.16 + 2155.63 = 4000.$

Intuitively, some readers may be wondering why, for example, member 2 (the $2000 contributor) would get more than twice as much as member 1 (the $1000 contributor). Asked differently, some readers may be wondering why member 2’s fair transfer-plan weight ($w_i$), 0.145278, would be more than twice as much as member 1’s fair transfer-plan weight ($w_i$), 0.066510.

Here, a slightly different example can help. Imagine a tontine fund with four otherwise identical 65-year-old men, except that while members 1, 2, and 3 each contribute $1000 to the tontine fund, member 4 contributes $3000. Now assume that member 1 dies, leaving members 2, 3, and 4 alive. Intuitively, it might seem that member 1’s $1000 contribution should be divided in
A tontine fund can fairly accommodate members with differing levels of contributions by using fair transfer–plan weights \((w_i)\) that take into account those different levels of contributions. There is one caveat, however: no one member can own more than half of the total risk of a tontine fund. Otherwise, the tontine fund could not provide that person with a fair bet for surviving the rest.\(^{124}\)

d. **Tontine Funds Can Properly Account for Investment Earnings**

In the simple tontine funds that we have considered so far, we have assumed that contributions do not earn any interest. In the real world, however, each member’s contributions would be invested, and each member’s balance would grow (or shrink) according to its investment performance. As members of a tontine fund die, mortality-gain distributions are based on the balance in each member’s account at the time of death.

We continue to use the variable \(s_i\) (which we have used so far only to signify member contributions) to denote the balance in member \(i\)’s account at any time \(t\); and, again, if the dying member is member \(j\), then each surviving member \(i\) would receive a mortality-gain distribution equal to \(s_j w_i / (1 - w_j)\), for \(i \neq j\). If the pool of tontine fund investors is large, then the deaths of members would occur relatively often, and each survivor would receive frequent payments of mortality-gain distributions that would continue until her own death.

\[^{124}\text{Here, a member’s risk means the product } f_i s_i \text{ of his force-of-mortality probability } (f_i) \text{ multiplied by his contribution } (s_i), \text{ and the total risk means the sum of all members’ risks. See Sabin, supra note 15, at 14 ("[A]n FTP exists if and only if no member holds more than half of the total risk of the pool."). Additional rules may be imposed that limit the total amount that a member may contribute. Id.}\]
Again, no member would ever be allowed to take any other distributions (i.e., no voluntary withdrawals). Once a contribution is made, it would remain in the tontine fund forever, along with any investment earnings. At the member’s death, the balance in the account would be distributed to the surviving members as mortality-gain distributions. This restriction is necessary because a member in failing health would otherwise seek to withdraw her contributions and the earnings on those contributions. Such “adverse selection” would invalidate the assumptions of the mortality table used to compute the fair transfer–plan weights ($w_i$).

e. Tontine Funds Could Also Take Increasing Longevity into Account

Finally, in the simple tontine funds we have considered so far, we have used the Social Security Administration's 2009 life tables. Over time, however, life expectancies are likely to increase, and these 2009 life tables will soon be out-of-date. Consequently, a real-world tontine fund should be designed to use the latest life tables so that it can make mortality-gain distributions based on the most recent death probability estimates.

2. Expected Benefits of Tontine Funds

We have shown the ease of designing a tontine fund that is fair to members of differing ages, genders, and contribution levels. To be sure, those who survive the longest would get better than average returns (i.e., mortality gains), while those who die young might not even recover their initial investments (i.e., mortality losses). On average, however, each member could expect to recover her initial contribution and any returns on that investment (less a modest management and recordkeeping fee).

Figure 1 shows a computer simulation of how a tontine fund with around 220 members might work. This simulation was designed by creating a tontine fund in which one new member joins each month. Each

---

125 See supra note 107 and accompanying text.
126 See supra Tables 1 & 2 (using the Social Security Administration's 2009 unisex and gender-based life tables, respectively).
128 As a legal matter, the tontine fund agreement would need to specify how and when it would choose a new life table for use in its fair transfer–plan.
129 See Sabin, supra note 15, at 24-25 (illustrating such a simulation).
new member’s gender was randomly selected, equiprobably male or female; each new member’s age was exactly 65; that is, his or her 65th birthday coincided with the joining date; and each member’s contribution was a randomly selected amount between $100 and $100,000. The number of members grows for several decades until it reaches an equilibrium of about 220 members, where, on average, one member dies each month, offsetting the new member who joins each month. Figure 1 shows the mortality gains that a typical long-lived male could expect after that equilibrium has been reached.

Figure 1: Normalized Mortality Gain from FTPs Versus Age for a Typical Long-Lived Male Member in a Simulated Tontine Fund

More specifically, Figure 1 plots the mortality-gain distributions paid to one of the longer-lived male members in the simulation (normalized to a contribution of $1). The plot began at the member’s joining age, age 65, and

---

130 Id. at 25 fig. 5.
ended at the time of his death. As the plot shows, benefits would be received at random times (i.e., when other members died) and in random amounts (i.e., varying with the contributions of the dying member). The average value of his benefit would increase with age, since the member’s own death probability \((q_i)\) and, consequently, his fair transfer-plan weight \((w_i)\) would increase with his age. In fact, it can be shown that the average value of a tontine fund member’s benefit depends only on that member’s age and gender (for \(q_i\)) and that member’s contribution \((s_i)\): the ages, genders, and contribution amounts of other members do not affect that member’s average benefit.\(^\text{131}\)

3. Two Problems with Tontine Funds

Two features of the tontine fund in Figure 1 stand out as serious negatives. First, mortality-gain distributions vary dramatically both in amount and timing, because they depend on when members die and how much those dying members had contributed. In short, payouts are noisy. Second, a member’s mortality-gain distributions start slow and low but increase rather dramatically at advanced ages, as the member’s death probability \((q_i)\) increases with age. In short, payouts are backloaded.

While the tontine fund always provides a fair bet to investors, these two disadvantages will discourage retirees from investing in them because, presumably, most retirees would prefer to have benefits that are level throughout retirement (like an immediate, level-payment annuity) or, alternatively, that increase gradually throughout retirement (like an immediate, inflation-adjusted annuity).

a. Reducing the Noisiness of a Tontine Fund

The noisiness of a tontine fund can be reduced by accumulating mortality-gain distributions over some period (e.g., a month), rather than paying them at the time of each member’s death, and by increasing the number of investors in the tontine fund. First, for example, a tontine fund can be designed to make monthly mortality-gain distributions as follows:

- Each member has an individual account that holds his contribution;

\(^{131}\) See id. at 5 (noting that, in a fair tontine, “a surviving member’s expected payout does not depend on the number of members in the pool, or the ages, genders, or contributions of the other members”).
When a member dies, the balance in his account is distributed to the accounts of the surviving members based on their respective fair transfer-plan weights \(w_i\); and

At end of each month, each living member receives a monthly mortality-gain distribution equal to the excess of the balance in his account over the amount of his initial contribution.

Second, increasing the number of members in a tontine fund would further decrease the noisiness of payouts. For example, imagine a tontine fund with approximately 5000 members of varying ages and genders who have made varying contributions. Again, for simplicity, assume that contributions do not earn interest. Table 4 shows a sample monthly statement for a member who had contributed $250,000 to a tontine fund and who lived through the month. More specifically, Table 4 shows that this member received a single distribution of $1041.67 at the end of the month, rather than varying amounts throughout the month (ranging from a low of $0 on most days to a high of $184.32 on April 7). In short, the noisiness of this tontine fund would be reduced through (1) making monthly mortality-gain distributions (rather than as each death occurs) and (2) having a large number of members in the pool (approximately 5000).

\[\text{132 In this example, two other members died on April 7, and this hypothetical member had $184.32 credited to her account ($184.32 = $135.41 + $48.91).}\]
Table 4: Sample Monthly Tontine Fund Statement for a Living Member

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount ($)</th>
<th>Balance ($)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/31</td>
<td>250,000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>04/02</td>
<td>67.17</td>
<td>250,067.17</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/03</td>
<td>25.21</td>
<td>250,092.38</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/05</td>
<td>55.14</td>
<td>250,147.52</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>135.41</td>
<td>250,282.93</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>48.91</td>
<td>250,331.84</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/12</td>
<td>52.29</td>
<td>250,384.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/15</td>
<td>102.54</td>
<td>250,486.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/20</td>
<td>159.46</td>
<td>250,649.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/21</td>
<td>139.68</td>
<td>250,785.82</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/22</td>
<td>17.82</td>
<td>250,803.63</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/25</td>
<td>124.81</td>
<td>250,928.44</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/28</td>
<td>55.32</td>
<td>250,983.76</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>57.91</td>
<td>251,041.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>(1041.67)</td>
<td>250,000.00</td>
<td>Payout of FTP Proceeds</td>
</tr>
</tbody>
</table>

In contrast, Table 5 shows the sample monthly statement for another member who is the same age and gender and contributed the same amount as the member in Table 4 but who died during the month. When she died on April 12, she forfeited the balance in her account on that date, and it was divided among the surviving members of the tontine fund (i.e., with the surviving member in Table 4 receiving $52.29 of the account on that date).  

---

133 This hypothetical tontine fund has approximately 5000 members of varying ages and genders who have made varying contributions. Mortality gains are based on a fair transfer-plan, and surviving members get a single payout at the end of the month.

134 In the real world, it would certainly take some time for the tontine fund manager to discover and record deaths and to compute the resulting mortality gains. Accordingly, actual monthly mortality-gain distributions might be delayed for a month or two. It would be more accurate to say that the surviving member in Table 4 is entitled to, and will eventually receive, the $52.29 attributable to the April 12th death of the member whose account is shown in Table 5.
Table 5: Sample Monthly Tontine Fund Statement for a Member Who Dies During the Month

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount ($)</th>
<th>Balance ($)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/31</td>
<td>250,000.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>04/02</td>
<td>67.17</td>
<td>250,067.17</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/03</td>
<td>25.21</td>
<td>250,092.38</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/05</td>
<td>55.14</td>
<td>250,147.52</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>135.41</td>
<td>250,282.93</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>48.91</td>
<td>250,331.84</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/12</td>
<td>(250,331.84)</td>
<td>0</td>
<td>Forfeited to FTP</td>
</tr>
</tbody>
</table>

Unfortunately, accumulating mortality-gains for monthly mortality-gain distributions and increasing the number of members in the tontine fund would do nothing to counteract the volatility that would invariably result from fluctuations in the value of the underlying investment assets. For example, if all of the tontine fund assets were invested in equities, then average monthly mortality-gain distributions could fall from, for example, $1000 a month for a typical member when the Dow Jones Industrial Average hit 14,000 (i.e., in October of 2007) to just $500 a month when the Dow Jones Industrial Average fell to 7000 (i.e., in February of 2009). To be sure, market fluctuations also play havoc with the prices and yields of traditional annuities and variable annuities that could be purchased by a retiree or by a pension plan. For example, if the market is down when a retiree decides to buy an annuity, she will only be able to buy a smaller annuity. Similarly, if interest rates are low when she decides to buy an annuity, the lifetime income stream that she purchases will also be low. Variable annuity payouts also vary with the performance of the underlying assets.

---

135 This hypothetical tontine fund has approximately 5000 members of varying ages and genders who have made varying contributions. Mortality gains are based on a fair transfer-plan, surviving members get a single payout at the end of the month, and dying members forfeit the balance in their accounts on the date of death. Dow Jones Industrial Average, supra note 54. Monthly mortality-gain distributions would also fluctuate with changes in the dividend and interest yields on the underlying assets. The Dangers of Buying an Annuity When Interest Rates Are Low, supra note 55. See Marotta, supra note 56 and accompanying text. We will further discuss how tontine financial products can help investors deal with market volatility in Section V.C, infra.
b. Reducing Backloading in a Tontine Fund

Unfortunately, it is impossible to reduce the backloading that is inherent in a tontine fund. The longer a member lives, the more she would receive, as her monthly mortality-gain distributions would generally increase with her age and her increasing death probability \((q_i)\).\(^{139}\) In the next Section, however, we will discuss how this backloading problem can be solved by adding an “annuity-payback mechanism.” The annuity-payback mechanism has the added benefit of further reducing the noisiness of the payouts. We call the resulting product a “tontine annuity.”

C. A Tontine Annuity

In this Section, we propose a tontine annuity that closely resembles a variable annuity. A tontine annuity is constructed by adding two enhancements to a tontine fund. First, as already discussed, to reduce noisiness, we would build in a monthly payment period; and, second, to eliminate backloading, we would add an annuity-payback mechanism.

1. Monthly Accrual of Fair Transfer-plan Payouts

In a tontine annuity, mortality-gain distributions would not be paid out immediately when other members die. Instead, mortality-gain distributions would be accrued within the individual accounts of the surviving members. If a member is alive at the end of the month, she would be paid the accrued mortality-gain distributions in her account as a monthly mortality-gain distribution (e.g., see Table 4). If she is not alive at the end of the month, she would receive nothing, as the balance in her account, including any mortality-gain distributions that accrued that month, would have been distributed to surviving members when she died during the month (e.g., see Table 5). Thus, a member would receive payments on a monthly schedule just as she would if she had instead purchased a variable annuity from an insurance company.

2. Annuity Payback

In addition to receiving a monthly mortality-gain distribution, each surviving member would also receive a portion of her original contribution at the end of each month that she is alive. Our approach is to make “monthly tontine-annuity distributions” to surviving members that are designed to

\(^{139}\) See Sabin, supra note 15, at 22-26; infra subsection II.C.2.
cancel out the age-related increase in mortality-gain distributions inherent in simple tontine funds like the one in Figure 1 (i.e., the backloading).

It turns out that a tontine annuity constructed in this way closely resembles an actuarially fair variable annuity (i.e., one without insurance agent commissions or insurance company reserves, risk-taking, and profits). To be sure, because the value of the assets in the tontine annuity fluctuates, monthly tontine–annuity distributions would still be volatile. But if we pretend that the underlying investment assets grow at a fixed, assumed rate of return, then the tontine annuity would provide monthly payouts that are approximately constant for life.

Moreover, it is relatively easy to determine the proper amounts of these monthly tontine-annuity distributions. The monthly payout of any actuarially fair annuity is simply equal to the account balance divided by a monthly annuity factor. The monthly annuity factor is the premium for an actuarially fair annuity that pays $1 per month for life. These monthly annuity factors can easily be calculated from a mortality table and depend only on the age of the annuitant and the assumed interest rate.¹⁴⁰

¹⁴⁰ This footnote explains how to compute a yearly annuity factor, which is the actuarial present value of a life annuity that pays $1 each year for life. The monthly annuity factor is approximately 12 times the yearly annuity factor.

We compute the annuity factor at each birthday by working backwards from the terminal age of the mortality table. For the 2009 Social Security Administration table that we use (see infra Appendix Table 1), the last entry is for age 119; thus the terminal age is 120, meaning that the table implies an individual always dies before her 121st birthday.

If the individual is alive at birthday 120, she receives $1. Since she does not survive to birthday 121, the only payment she receives is the single dollar at age 120, so the actuarial present value of the annuity is $1. Thus:

\[ a_{120} = 1. \]

If the individual is alive at birthday 119, she receives $1. In addition, if she survives to birthday 120, she will receive a future payment stream having an actuarial value of \( a_{120} \). Thus, at birthday 119, the actuarial present value of payments is

\[ a_{119} = t + (1 - q_{119}) \times a_{120} / (1 + d), \]

where \( q_{119} \) is the probability of dying during age 119 (i.e., before birthday 120), which is given in the mortality table; and \( d \) is the discount rate (e.g., \( d = .07 \), or 7%).

Similarly, if the individual is alive at birthday 118, she receives $1, and if she survives to birthday 119, she will receive a future payment stream having an actuarial value of \( a_{119} \). Thus, at birthday 118, the actuarial value of payments is:

\[ a_{118} = t + (1 - q_{118}) \times a_{119} / (1 + d). \]

Continuing in this manner, we calculate the annuity factor \( a_{117} \) for birthday 117, \( a_{116} \) for birthday 116, and so on, until we reach the birthday of interest. For example, for the 2009 Social Security Administration table and a discount rate of 7%, continuing until birthday 65 gives \( a_{65} = 10.359 \). (That is, the actuarial present value of an annuity that pays $1 each year for the life of a
For example, Table 6 shows a sample monthly statement for a member of a tontine annuity who lives through the first month after turning age 65 and who had exactly $250,000 in his account at the end of the prior month. The only difference between the monthly statement in Table 4 and the monthly statement in Table 6 is that instead of receiving a monthly mortality-gain distribution of just $1041.67 (as in Table 4), our hypothetical member would receive a monthly tontine-annuity distribution of $2133. That $2133 is computed by dividing the account balance on the last day of the month (i.e., $251,041.67 on April 30th) by the applicable monthly annuity factor (i.e., 117.6939). That is, the monthly tontine-annuity distribution for the just-turned-65-year-old member in Table 6 is $2133 ($2133.00 = $251,041.67/117.6939).

65-year-old is $10.36 (at a 7% discount rate.) As mentioned, the monthly annuity factor is approximately 12 times the yearly annuity factor, and Column 5 of Appendix Table 1, infra, shows that the monthly annuity factor for the first month of the year in which our hypothetical retiree turns 65 is 117.6939, or about 12 x 10.359.

141 Column 5 of Appendix Table 1, infra, shows the applicable monthly annuity factors for the first month of each year starting with age 65, when monthly tontine-annuity distributions are expected to commence.
Table 6: Sample Monthly Tontine Annuity Statement for a Living Member
(for the First Month After the Member Turned 65)\textsuperscript{142}

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount</th>
<th>Balance</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/31</td>
<td>250,000.00</td>
<td>250,000.00</td>
<td></td>
</tr>
<tr>
<td>04/02</td>
<td>67.17</td>
<td>250,067.17</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/03</td>
<td>25.21</td>
<td>250,092.38</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/05</td>
<td>55.14</td>
<td>250,147.52</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>135.41</td>
<td>250,282.93</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>48.91</td>
<td>250,331.84</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/12</td>
<td>52.29</td>
<td>250,384.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/15</td>
<td>102.54</td>
<td>250,486.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/20</td>
<td>159.46</td>
<td>250,649.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/21</td>
<td>139.68</td>
<td>250,785.82</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/22</td>
<td>17.82</td>
<td>250,803.63</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/25</td>
<td>124.81</td>
<td>250,928.44</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/28</td>
<td>55.32</td>
<td>250,983.76</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>57.91</td>
<td>251,041.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>(2133.00)</td>
<td>248,908.67</td>
<td>Tontine-annuity Distribution</td>
</tr>
</tbody>
</table>

Alternatively, a tontine annuity could be designed to make monthly tontine-annuity distributions that mimic an inflation-adjusted variable annuity. That inflation-adjusted tontine annuity would make lower monthly tontine-annuity distributions in the early years but greater distributions for those who live to later years. For example, if inflation is assumed to be 3% per year, then the first monthly tontine-annuity distribution for the hypothetical 65-year-old in Table 6 would be just $1651.72 ($1651.72 = $251,041.67/151.9876),\textsuperscript{143} but distributions in subsequent months would be

\textsuperscript{142} This hypothetical tontine annuity has approximately 5000 members of varying ages and genders who have made varying contributions. Mortality gains are based on a fair transfer-plan, and surviving members get a single payout at the end of the month, based on the applicable monthly annuity factor.

\textsuperscript{143} Column 6 of Appendix Table 1, infra, shows the inflation-adjusted applicable monthly annuity factors for the first month of each year starting with age 65, when monthly tontine-annuity distributions are expected to commence.

This footnote explains how to compute a yearly inflation-adjusted annuity factor, which is the actuarial present value of a life annuity that pays $1 the first year and then increases future annual payments by the assumed inflation rate. The monthly annuity factor is approximately 12 times the yearly annuity factor.

The annuity factor is computed in a manner similar to the uniform case in note 140, supra, except that now it includes the inflation adjustment. Letting $i$ denote the inflation rate (e.g., $i = .03$ or $3\%$), then:
larger and would eventually exceed the payout level of the not-adjusted-for-
inflation tontine annuity.

In short, a tontine annuity could be designed to resemble an actuarially
fair variable annuity or an actuarially fair inflation-adjusted variable annuity.
These tontine annuities would still be volatile because of fluctuations in the
value of the underlying investment assets, but backloading would be
eliminated.

3. Adding in Investment Income

In the simple tontine annuities we have considered so far, we have
assumed that contributions do not earn any interest. In the real world,
however, each member's contributions would be invested, and the member's
balance would grow (or shrink) according to its investment performance.
Accordingly, account balances at the end of each month would tend to be
higher, and monthly tontine-annuity distributions would also tend to be
higher. For example, if the tontine annuity in Table 6 had earned $1000 of
investment interest in that month, the balance in the account at the end of
the month would have been $1000 higher, and, consequently, the monthly
tontine distribution would have been $8.52 higher—$2141.52 instead of the
$2133, as shown in Table 6 ($2141.52 = 2133 + 8.52$).

\[
a_{t+1} = \frac{a_t \cdot (1 + i) \cdot (1 - q_{t+1}) \cdot (1 + d)}{1 + d},
\]

and so forth. For example, we can show that if the inflation parameter is set to 3%, then \( a_{65} = 13.216 \).

As mentioned, the monthly annuity factor is approximately 12 times the yearly annuity factor,
and Column 6 of Appendix Table 1, infra, shows that the inflation-adjusted monthly annuity factor
for the first month of the year in which our hypothetical retiree turns 65 is 151.9876, or about 12 \times 13.216.

\[^{144}\text{See infra Table 7.}\]
Table 7: Sample Monthly Tontine Annuity Statement for a Living Member, with Investment Earnings (for the First Month After the Member Turned 65)\textsuperscript{145}

<table>
<thead>
<tr>
<th>Date</th>
<th>Amount</th>
<th>Balance</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/31</td>
<td>250,000.00</td>
<td>250,000.00</td>
<td></td>
</tr>
<tr>
<td>04/02</td>
<td>67.17</td>
<td>250,067.17</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/03</td>
<td>25.21</td>
<td>250,092.38</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/05</td>
<td>55.14</td>
<td>250,147.52</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>135.41</td>
<td>250,282.93</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/07</td>
<td>48.91</td>
<td>250,331.84</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/12</td>
<td>52.29</td>
<td>250,384.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/15</td>
<td>102.54</td>
<td>250,486.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/20</td>
<td>159.46</td>
<td>250,649.13</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/21</td>
<td>139.68</td>
<td>250,785.82</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/22</td>
<td>124.81</td>
<td>250,908.63</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/28</td>
<td>55.32</td>
<td>250,983.76</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>57.91</td>
<td>251,041.67</td>
<td>Proceeds from FTP</td>
</tr>
<tr>
<td>04/30</td>
<td>1000.00</td>
<td>252,041.67</td>
<td>Interest for the month</td>
</tr>
<tr>
<td>04/30</td>
<td>(2141.52)</td>
<td>249,900.15</td>
<td>Tontine-annuity Distribution</td>
</tr>
</tbody>
</table>

4. Managing Investments

Investments in a tontine annuity would most likely be managed collectively for the entire pool, but it would be possible to design a tontine annuity which allows members to direct their own investments, just as people often do with their self-directed 401(k) plans and IRAs.\textsuperscript{146} Pertinent here, rates of return are likely to be much higher if the investments are

\textsuperscript{145} This hypothetical tontine annuity has approximately 5000 members of varying ages and genders who have made varying contributions. Mortality gains are based on a fair transfer-plan, and surviving members get a single payout at the end of the month based on the applicable monthly annuity factor.

\textsuperscript{146} Of course, default investments could be offered to individual investors, just as target date funds are typically a default investment offered in self-directed 401(k) plans. See U.S. DEP’T OF LABOR, EMP. BENEFITS SEC. ADMIN., TARGET DATE RETIREMENT FUNDS—TIPS FOR ERISA PLAN FIDUCIARIES (2013), available at http://www.dol.gov/ebsa/pdf/fsTDF.pdf (providing guidance to fiduciaries of 401(k) and other employee-directed retirement programs regarding selecting and monitoring target date retirement funds).
managed by professionals rather than allowing individuals to direct their own investments.\textsuperscript{147}

In theory, a tontine annuity could be managed by a discount broker, and no money would have to be set aside for insurance agent commissions or insurance company reserves, risk-taking, or profits. Those commercial insurance charges can be quite hefty.\textsuperscript{148} For example, a recent Morningstar survey of 2037 variable annuities showed an average administrative fee in 2014 of 1.33\% of assets under management, and that fee is on top of the cost of managing the underlying investments, which itself can easily run another 1.0\%.\textsuperscript{149} To be sure, some discount brokers have recently teamed up with

\begin{footnotesize}
\begin{enumerate}
\item Indeed, experts estimate that the typical commercial life annuity has a 12\% "load" factor due to the combination of administrative expenses and adverse selection; that is, the typical commercial life annuity provides benefits that are worth just 88\% of an actuarially fair annuity (i.e., a "money's worth ratio" of 88\%). See MARK J. WARSHAWSKY, RETIREMENT INCOME: RISKS AND STRATEGIES 66 (2012) ([D]ue to a combination of administrative costs and selection effects, the nominal annuity is assumed to have a money's worth ratio of 0.88, that is, the couple faces a 12 percent load factor on their annuity purchase.). Put differently, the payouts from actuarially fair annuities would be around 15\% higher than in current annuity markets. See James Poterba et al., The Composition and Drawdown of Wealth in Retirement, J. ECON. PERSP., Fall 2011, at 102 tbl.3 (providing that the actuarially fair life annuity for a 65-year-old-man in 2008 was 9.95\% and the AnnuityShopper price for a commercial life annuity was just 8.46\%, thus indicating a load factor of 17.6\%: 9.95\%/8.46\% - 1 = 17.6\%); see also Jeffrey R. Brown et al., The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program ([T]he expected present value of annuity payouts is typically below the purchase price of the annuity . . . . ), in RISK ASPECTS OF INVESTMENT-BASED SOCIAL SECURITY REFORM 321, 321-22 (John Y. Campbell & Martin Feldstein eds., 2001); James M. Poterba & Mark Warshawsky, The Costs of Annuitizing Retirement Payouts from Individual Accounts ("The cost of such annuities, including both administrative and sales costs, the 'adverse selection' costs associated with voluntary purchase behavior, and return on capital for the insurance company, is typically below the purchase price of the annuity . . . . "); see also INSURED RET. INST., 2011 IRI FACT
\end{enumerate}
\end{footnotesize}
insurance companies to offer low-cost variable annuities. For example, Charles Schwab & Co., Inc., markets variable annuities with insurance charges that range from 0.60% to 0.65% (again, not including the additional administrative expenses involved in managing the investments), and The Vanguard Group, Inc. offers a variable annuity with an insurance charge of 0.57%. Again, these insurance charges do not include the additional administrative expenses involved in managing the underlying investments.

We are confident that discount brokers would be able to offer tontine annuities at even lower costs. As there are no insurance guarantees associated with tontine annuities, we believe that discount brokers could offer these products with total annual costs, perhaps, as low as 0.30% of assets under management, depending on the nature of the underlying investments. That means retirees would get significantly more benefits than they do with today’s high-cost variable annuities. For example, imagine a tontine annuity that invested entirely in an S&P 500 stock index fund. We know that most discount brokers offer an S&P 500 index fund with expense ratios of 0.10% or less, and we believe that the tontine annuity management and recordkeeping functions could be performed for as little as 0.20% of assets under management. That means total costs could be as low as 0.30% of assets under management.

BOOK 56 figs.3-5 (2011), available at http://www.advisorsexcel.com/downloads/2011FactBook.pdf (showing that average total expenses for variable annuities in 2010 were 2.33%, compared with average total expenses for mutual funds that year of just 1.32%). The additional expenses associated with variable annuities include both so-called “mortality and expense” (M&E) charges and separately stated administrative expenses.

150 CHARLES SCHWAB, supra note 149.

151 The Vanguard Group, Inc. offers a variable annuity with a total expense ratio ranging from 0.46% to 0.77%. Tax-Deferred Retirement Savings with the Vanguard Variable Annuity, VANGUARD, https://investor.vanguard.com/annuity/variable (last visited Jan. 16, 2015), archived at http://perma.cc/YU8C-2UV6.

In that regard, TIAA–CREF Financial Services has been offering a low-cost, tontine-like product for years. Created in 1952, the College Retirement Equities Fund (CREF) was the world’s first variable annuity. Today, CREF operates eight investment accounts that differ by objective: stocks, bonds, money market, and social choice; and CREF keeps its costs for managing those accounts at between 0.395% and 0.465% of assets under management. CREF participants choose which fund to invest in; and later on, they choose from among a variety of distribution options, including one-life and two-life annuities. When a retiree selects a life annuity, the annuity payments will depend on both the investment experience of the chosen accounts and on the mortality experience of the other participants. Basically, within each investment account, CREF periodically adjusts the annuity payments so that the present value of the aggregate amount expected to be paid out over the participants’ remaining lifetimes matches the current value of the assets in the account. If participants in the fund “live longer . . . than expected, the amount payable to each will be less than if they as a group die sooner than expected.” In short, like a tontine, the mortality risk falls on the annuitants and is not guaranteed by CREF (or TIAA).

154 See also Id.; see also Poterba & Warshawsky, supra note 148, at 191-98 (discussing the history and development of individual annuities offered by TIAA–CREF).
157 Id. at 74-75. Of note, TIAA–CREF annuities have been using unisex life tables since 1982. See Spirt v. Teachers Ins. & Annuity Ass’n, 691 F.2d 1054, 1066 (2d Cir. 1982) (holding that TIAA–CREF is subject to Title VII, thus forbidding the use of sex-based mortality tables to calculate benefits based on contributions), vacated on other grounds, 463 U.S. 1223 (1983).
158 But see TIAA–CREF FIN. SERVS., supra note 156, at 76 (mentioning that mortality experience has “not historically had a significant impact”).
159 Id. at 73. For more details, see generally TIAA–CREF FIN. SERVS., COLLEGE RETIREMENT EQUITIES FUND (“CREF”) SUPP. NO. 1 B-41 to B-42 (2014), available at http://www1.tiaa-cref.org/public/prospectuses/cref_sai.pdf.
160 TIAA–CREF FIN. SERVS., supra note 156, at 73. Of note, rather than using a fair transfer-plan to share mortality gains from each dying member (as our tontine annuity would), CREF’s method shares aggregate mortality gains and losses. Consequently, some participants will get a better deal, and some will get a worse deal than they would with a fair transfer-plan. Cf. Sabin, supra note 15, at 59-62 (discussing the bias present in group self annuities that give some members better payouts than others).
As mentioned, tontines were popular at the end of the nineteenth century, but they fell out of favor at the beginning of the twentieth century, largely due to fraud and mismanagement of early tontine funds. In today’s post-ERISA world, however, it would be relatively easy for the U.S. Securities & Exchange Commission (SEC) and the U.S. Department of Labor’s Employee Benefits Security Administration to regulate tontine annuities and the fiduciaries that would manage them. Moreover, private sector recordkeepers and custodians would help protect tontine annuity assets.

We live in an era in which new financial and lifetime income products are created all of the time. Indeed, GLWB funds were developed in Canada only recently, before spreading to the United States and other countries, and as mentioned, a number of discount brokers have recently teamed up with insurance companies to offer low-cost variable annuities. Accordingly, we anticipate that a number of discount brokers and insurance companies

Also, while our tontine pension (discussed infra Section II.D) results in forfeitures by workers as well as retirees, CREF participants do not face any forfeitures at all until participants voluntarily elect to take their distribution in the form of a one-life or two-life annuity, and typically such elections are not made until retirement after age 59 ½. TIAA–CREF FIN. SERVS., supra note 156, at 72-75.


161 See supra text accompanying notes 12-14.
162 See Milevsky & Shao, supra note 95, at 50, 56 (discussing the creation of GLWB products in Canada and their subsequent spread to the United States).
163 See supra text accompanying notes 150-151.
will want to develop new tontine annuity products and seek the regulatory approvals that might be needed.

5. Adverse Selection Is Always a Challenge for Annuities

To be sure, underutilization would be a problem for tontine annuities, just as it is for traditional annuities. All in all, as more fully explained below, people rarely choose to buy annuities voluntarily. In fact, over the years, there has been a significant decline in the annuitization of retirement savings by American workers. The shift from traditional defined benefit plans to defined contribution plans is a large part of the story, as defined contribution plans typically distribute benefits in the form of lump sum distributions rather than as annuities. Indeed, relatively few defined contribution plans even offer annuity options, and, in any event, not many participants elect those annuity options. In short, the demand for annuities is lower than expected, a shortfall which has come to be known as the “annuity puzzle.”


165 TOwers wATSON, supra note 48, at 15.


167 See, e.g., Shlomo Benartzi et al., Annuity Puzzles, J. ECON. PERSP., Fall 2011, at 143, 154-57 (discussing behavioral and institutional factors leading to the low demand for annuities); Franco Modigliani, Life Cycle, Individual Thrift, and the Wealth of Nations, 76 AM. ECON. REV. 297, 307 (1986) (“[I]t is a well-known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare.”). See generally Menahem E. Yaari,
There are many reasons for this low demand for annuities, but adverse selection is one of the most important reasons.\textsuperscript{168} Basically, those who voluntarily purchase annuities tend to live longer than those that do not, and, consequently, annuities are not priced very well for those with normal life expectancies.\textsuperscript{169}

a. \textit{Adverse Selection and Tontine Annuities}

Adverse selection would also be a problem for tontine annuities. Just as the people who voluntarily purchase traditional annuities tend to live longer than those that do not, people who would choose to invest in a tontine annuity would tend to live longer than those who would not. To be sure, the tontine annuity would offer a better expected return than a commercial variable annuity, but coverage would nevertheless be skewed towards longer-lived investors. In short, as with traditional annuities, tontine annuities would be underutilized.

b. \textit{Solving the Adverse Selection Problem}

In general, problems with adverse selection are solved with broad coverage.\textsuperscript{170} For example, group health insurance premiums are low for large employers: they can generally ignore adverse selection as long as they provide healthcare coverage for virtually all of their employees. Similarly, Social Security and large defined benefit plan pensions can generally ignore adverse selection because they cover large numbers of employees. In short, the solution to adverse selection is to cover a broad group of individuals, and in the next Section, we show how a large employer could overcome the adverse selection against tontine annuities by adopting a “tontine pension” for a large group of its employees.


\textsuperscript{169} See MACKENZIE, supra note 168, at 43 (explaining that, in the life annuities market, moral hazard would lead to healthier behavior, meaning annuitants would tend to engage in behaviors increasing their lifespan).

\textsuperscript{170} See id. at 41 (“Universal mandatory annuitization of part or all of the balances in individual accounts would lower the average life expectancy of the annuitant population, and should lower the average premium for each sex.”).
D. Tontine Pensions

While tontine annuities would be attractive investments in their own right, they are likely to be as underutilized as traditional annuities and other lifetime income products. Individual investors generally underestimate their life expectancies and shy away from annuities and other lifetime income products. That is where pensions come in. Just as group health insurance spreads health risks over large groups, traditional defined benefit pension plans spread longevity risk over large groups: traditional pensions either provide annuity-like retirement benefits to their participants or purchase group annuities for them.

Unfortunately, as we have seen, traditional defined benefit pensions in both the private and public sector are often underfunded and, in recent years, we have seen numerous plan sponsors freeze, terminate, or replace their plans. Market volatility, shrinking labor forces, and increasing life expectancies have all exerted pressure on traditional defined benefit plans and their sponsors. It is no wonder that we have seen defined contribution plans supplant defined benefit plans in the private sector, and there is increasing pressure on public employers to also consider replacing their traditional defined benefit plans with defined contribution plans. For example, 50% of full-time private industry workers in the United States participated in defined contribution plans in 2011, up from 40% in 1989–1990; meanwhile, participation in defined benefit plans fell from 42% in 1989–1990 to just 22% in 2011. All in all, the era of the traditional defined benefit plan is largely over.

---

171 See supra subsection II.C.5.
172 See supra subsection I.B.2.a.
173 See supra text accompanying notes 42-45.
174 Id. Pertinent here, for example, the Pension Benefit Guaranty Corporation took over "111 newly failed single-employer plans" in Fiscal Year 2013. PENSION BENEFIT GUAR. CORP., supra note 43, at 5. Further, the City of Detroit went into bankruptcy in large part because of its pension debts. See Monica Davey et al., Detroit Ruling Lifts a Shield on Pensions, N.Y. TIMES, Dec. 4, 2013, at A1 (discussing a bankruptcy judge's finding that Detroit public employees' pensions were not protected in a bankruptcy).
175 WIAŁTROWSKI, CHANGING LANDSCAPE OF EMPLOYMENT-BASED RETIREMENT BENEFITS, supra note 164. More specifically, there were 683,000 private pension plans in 2011. U.S. DEPT. OF LABOR, EMP. BENEFITS ADMIN., PRIVATE PENSION PLAN BULLETIN 1 (2013), available at http://www.dol.gov/ebsa/PDF/2011pensionplanbulletin.PDF. These are ERISA-covered plans and do not include non-ERISA plans such as IRAs and Roth IRAs. Of these ERISA-covered plans, just 45,756 were defined benefit plans (with 40.9 million participants and $2.5 trillion in assets), while 638,390 were defined contribution plans (with 88.7 million participants and $3.8 trillion in assets). Id. at 3 tbl.A1. Of these defined contribution plans, 523,000 were 401(k)-type plans. Id. at 2.
That is where tontine pension plans can come in. Like a typical defined contribution plan, a tontine pension would always be fully funded. Like a traditional defined benefit plan, however, a tontine pension would make annuity-like payments for as long as its retirees lived. This Section explains how a tontine pension would work.

1. A Simple Tontine Pension

An employer who wanted to provide a tontine pension for its employees would set up a defined-contribution-style pension plan, only instead of investing its contributions in stocks and bonds, the employer would invest in a tontine annuity for its employees. For example, each year, an employer might make contributions of 10% of its employees’ salaries. Those contributions would be held in trust and invested in a tontine annuity, and allocated to the individual tontine pension accounts of the participants. The difference is largely in the payouts. Rather than being able to receive lump sum distributions (or periodic payments or a life annuity), each tontine pension plan participant would receive benefits based on the tontine principle. That is, the employer contributions for each participant, and the investment earnings on those contributions, would be held in a tontine annuity, and the “monthly tontine-pension distributions” would be the only kind of distributions made to retirees.


More specifically, starting at the participant’s normal retirement age (or later, if she so elected), the balance in her tontine pension account would be paid out to her in the same manner as if she had purchased her own tontine annuity with the employer contributions made on her behalf. No other form of distribution would ever be permitted. For example, for a typical worker who had accumulated $250,000 at her retirement, her monthly statement would look just like the sample monthly statement for the tontine annuitant in Table 7.

In short, a tontine pension would provide lifetime retirement income in a way similar to a defined contribution platform. Essentially, the tontine pension is like a defined contribution plan that only pays benefits in the form of an actuarially fair life annuity. The difference is that rather than having the plan sponsor purchase annuities for each retiring employee or otherwise bear the risks and costs of providing the promised annuity benefits, with a tontine pension, the plan sponsor bears no investment or actuarial risks at all. The tontine pension would make distributions to retirees out of the funds accumulated in the underlying tontine annuity and in accordance with the fair transfer-plan and annuity-payback protocols. These monthly tontine-pension distributions could be designed to mimic immediate, level-payment annuities; immediate, inflation-adjusted annuities; deferred annuities; or joint and survivor annuities.

2. Tontine Pensions Compared with Other Pension Alternatives

a. Tontine Pensions Versus Traditional Defined Benefit Plans

A tontine pension could easily be designed to pay benefits that were, on average, comparable to those paid by a traditional, final-average-pay defined benefit plan. To be sure, the benefits paid by a tontine pension would vary from month to month because of fluctuations in the value of the underlying assets and the variability inherent in the indeterminateness of the deaths of

---

177 See infra Section III.C.
178 Id.
179 We note that a tontine pension is basically a kind of deferred annuity. For example, unless an unmarried participant survives until retirement, she would forfeit the balance in her tontine pension account (just like an unmarried participant in a traditional defined benefit plan). If she wanted to defer her payouts even longer, for example, until age 85, then her account would simply reinvest the mortality-gain distributions from dying participants until that time. Because of adverse selection, it might be necessary for such deferral elections to be made years in advance.
180 For more on how to design such qualified joint and survivor tontine annuities, see infra subsection V.D.3.
other participants in the tontine pension. But, on average, benefits paid by a tontine pension would approximate an actuarially fair life annuity.

With a defined benefit plan, the variation in monthly payments is eliminated, but only because the plan sponsor (the employer) guarantees the promised payments. The plan sponsor bears all the contribution, mortality, and investment risks, and we have, of course, seen how poorly that has worked out, with thousands of failed plans in the private sector and numerous underfunded plans in both the private and public sectors.\footnote{See supra notes 42-45 and accompanying text.} While plan sponsors do a much better job growing investments than individuals,\footnote{See, e.g., Forman & Mackenzie, supra note 29, at 6-39 to 6-40 (“[T]raditional defined benefit plans generally outperform [individually managed] 401(k) plans.”); Forman, supra note 147, at 9-5 (noting that there were “numerous economies of scale associated with traditional pension plans”); Munnell et al., supra note 147, at 6 (“Preliminary data suggest that IRAs underperform employer-sponsored plans.”).} plan sponsors do not always have the discipline to make the contributions that are needed to keep their traditional defined benefit plans fully funded.\footnote{See supra notes 42-45 and accompanying text.} On the other hand, tontine pensions would always be fully funded, just as defined contribution plans are almost always fully funded—through regular contributions equal to, for example, 10% of salary.\footnote{To be sure, employers sometimes cut their contribution rates to defined contribution plans, but such plans are still fully funded by the contributions that are made.}

In short, tontine pensions have two major advantages over traditional defined benefit plan pensions. First, unlike traditional pensions which are frequently underfunded, tontine pensions would always be fully funded. Second, unlike traditional pensions where the plan sponsor must bear all the investment and actuarial risks, with a tontine pension, the plan sponsor bears neither of those risks.

b. Tontine Pensions Versus Typical Defined Contribution Plans

So how do tontine pensions stack up against typical defined contribution plans? The answer is very well, indeed. Like a typical defined contribution plan, a typical tontine pension might start with employer contributions equal to, for example, 10% of salary. In the typical defined contribution plan, however, the participants are often allowed to direct the investment of their individual accounts, and payouts almost always take the form of lump sum and periodic distributions, rather than life annuities.\footnote{See supra note 48 and accompanying text.} On the other hand, with a tontine pension, the plan sponsor could, and should, manage the
investments, and benefits would be paid out only as a tontine pension that approximates an actuarially fair variable annuity.

To be sure, a plan sponsor could design a defined contribution plan where the plan sponsor manages all the investments and where benefits are only paid out in the form of a life annuity. But we know of no defined contribution plans like that, and we doubt that any employer with a defined contribution plan would have the discipline to design and continue such a plan in the face of employee expectations and demands (1) that the employees be allowed to direct their investments and (2) that the employees be allowed to receive the balance in their accounts as periodic or lump sum distributions rather than only as life annuities.

In fact, we believe that a tontine pension is reasonably analogous to a defined contribution plan with mandatory annuitization. There are a couple of key differences, however. First, with a tontine pension, those who survive until retirement would also benefit from the forfeitures of the accounts of those who did not. As far as we know, that does not happen with any defined contribution plans. Second, while a tontine pension would automatically provide benefits that approximate an actuarially fair life annuity, a defined contribution plan would have to purchase a lower-yielding commercial annuity to provide a mandatory annuitization benefit.

c. Tontine Pensions Versus Cash Balance Plans

A tontine pension is also similar to a cash balance plan with mandatory annuitization. In a cash balance plan, the sponsor credits hypothetical individual accounts with contributions of, for example, 10% of compensation. As with traditional defined benefit plans, the default benefit in a cash balance plan is a life annuity; however, cash balance plans typically allow lump sum and periodic distributions as well.\textsuperscript{186} Indeed, we doubt that there are many cash balance plans that require benefits be taken in the form of a life annuity, and we doubt that there are many employers that would have the discipline to design or to continue such a plan in the face of employee expectations and demands that the employees be allowed to receive the balance in their accounts as periodic or lump sum distributions rather than only as annuities.

Moreover, because cash balance plans are defined benefit plans, like traditional pensions, cash balance plans are often underfunded.\footnote{See Kevin Olsen, PBGC Sues to Take Over Dewey & LeBoeuf Retirement Plans, PENSIONS & INVESTMENTS (May 15, 2012), http://www.pionline.com/article/20120515/ONLINE/120519943/pbgc-sues-to-take-over-dewey-amp-leboeuf-retirement-plans, archived at http://perma.cc/L5ZT-UTFW (describing an example of an underfunded cash balance pension plan).} On the other hand, with a tontine pension, the plan sponsor’s contributions would be fixed at, for example, 10% of compensation, and the plan would then be fully funded with those actual contributions. The plan sponsor would then manage and grow the investments, and the tontine-pension distributions would approximate an actuarially fair life annuity.

3. Summary of the Advantages and Disadvantages of Tontine Pensions

In essence, a tontine pension would be like a traditional defined benefit pension plan, except that it would always be fully funded and the plan sponsor would never bear any of the investment or actuarial risks. Participants would receive monthly tontine pension benefits for as long as they lived, and a tontine pension could be designed to provide inflation-adjusted annuities, deferred annuities, or joint and survivor annuities.\footnote{See supra notes 177-180 and accompanying text.} Conceivably, individual participants could be allowed to make additional elective contributions to their accounts, just as they do now under 401(k)-type plans.\footnote{But see infra subsection V.D.2 (providing reasons why participants might be reluctant to make such contributions).}

The principal disadvantage of a tontine pension is that monthly payments would vary in amount. One source of variation is the randomness of member deaths, but the more individuals who participate in the plan, the less significant that noisiness would be. For a tontine pension that covers thousands of participants, the variation due to random deaths would be minimal.\footnote{See supra subsection II.B.3.a.} However, there could still be considerable variation due to volatility in both the value of the underlying assets and the rate of return on those assets.\footnote{See supra note 136 and accompanying text.}

Finally, as with traditional defined benefit plans, participants who live the longest would collect the most benefits, and those who died young might not even recover the amounts contributed on their behalf. Of course, that is the nature of traditional defined benefit plans, life annuities, and most other lifetime income products, so it is not a “disadvantage" unique to tontine pension plans.
III. MODELING A SIMPLE TONTINE PENSION

In this Part, we design a model tontine pension for a large employer and then use a computer simulation to see what kinds of tontine pension benefits the participants could expect to receive.

A. The Parameters of the Simulation

Our computer simulation uses a pool of approximately 170,000 members (approximately 100,000 active employees and 70,000 retirees). The parameters of the simulation are as follows:

- The employer hires 3600 employees each year (300 each month).
- The employee's gender is randomly selected, equiprobably male or female.
- Each employee is hired on her 35th birthday and works continuously for the employer for 30 years until age 65, or earlier death.\(^{192}\)
  - Each employee is hired at a salary of $50,000 a year, and her salary increases 4.0% each year.\(^{193}\)
  - At retirement, each employee receives a tontine pension until death.
    - In this simple simulation, nobody is married (so no joint and survivor annuity benefits are needed).
    - The account balances of those who die are forfeited.\(^{194}\)
- Every year, the employer contributes 10% of salary for every employee to the tontine pension.\(^{195}\)

\(^{192}\) We chose 30 years as a reasonable career with the employer. Obviously, workers who work 35 years would earn proportionally more tontine pension benefits, and those who work 25 years would earn proportionately less benefits. Tontine pension benefits would also vary if workers started working before or after our assumed start age of 35 or retired before or after our assumed retirement age of 65.

\(^{193}\) In that regard, for example, the CalSTRS defined benefit plan uses a 3.75% annual wage growth assumption. MILLIMAN, supra note 19, at 57 tbl.B.1.

\(^{194}\) If we had assumed that living workers could leave, their account balances would go with them to their new employer’s plan, and vice versa, so we ignore them.

\(^{195}\) We use the very plausible 10% contribution rate. That rate has the added advantage that it is easy to extrapolate away from it. For example, if one thinks that 15% is a better contribution rate, one need only multiply most of our model’s results by 150%. Nor must the contributions
Investment return: funds are professionally managed and earn 7.0% net of investment expenses each and every year, compounded annually.\textsuperscript{196}

Inflation is 3.0% each year.\textsuperscript{197}

Workers receive no payouts until age 65,\textsuperscript{198} and then retirees receive either uniform (fixed) annuity-type payouts or, alternatively, inflation-adjusted annuity-type payouts.\textsuperscript{199}

The mortality model is based on the Social Security Administration 2009 unisex mortality table.\textsuperscript{200}

Therefore, at equilibrium, approximately 3000 out of the 3600 initial hires each year reach age 65; approximately 100,000 are actively employed at any time; and there are approximately 70,000 retirees at any point in time.

B. Calculation of the Retirement Balance

At the outset, Table 8 shows how this tontine pension would work for workers ages 35 through 64. Column 1 of Table 8 shows the age of each worker from ages 35 through 64. Column 2 shows the salary of that worker each year. Column 3 shows the amount of the 10%-of-salary contribution that her employer makes to the tontine pension on her behalf each year.

\textsuperscript{196} Our 7.0% investment return assumption is also fairly reasonable. For example, the CalSTRS defined benefit plan uses 7.5% as its estimate of investment return (net of investment and administrative expenses). MILLIMAN, supra note 19, at 57 tbl.B.1. While many public pension plans have even higher assumed rates of return and have historically achieved those higher rates of return, many analysts believe we are in a low return environment for the indefinite future. See James J. Rizzo & Piotr Krekora, Presentation on the Goldilocks Principle & Investment Return Assumptions at Florida Government Finance Officers Association 2013 Annual Conference 41 (June 25, 2013), available at http://www.fgfoa.org/Assets/Files/Jim_Rizzo_Presentation_PDF.pdf (finding that 6.78% was the average rate of return projected by 8 national investment consulting firms for public pension plan portfolios over the next 15 years, compared with the 8% rate of return that those plans commonly assume).

\textsuperscript{197} For example, the CalSTRS defined benefit plan uses a 3.0% inflation assumption. MILLIMAN, supra note 19, at 57 tbl.B.1.

\textsuperscript{198} To make the simulation less complicated, only the retirement phase (i.e., the payouts to those age 65 and older) was simulated. The account balance at age 65 was set equal to the expected value (i.e., the statistical average) of the account of a worker who survives to age 65. The number of workers surviving to retirement was set to its expected value from the Social Security Administration’s 2009 unisex life table. Bye, supra note 109.

\textsuperscript{199} That is, the expected value of payouts is either uniform or inflation-adjusted.

\textsuperscript{200} Bye, supra note 109.
Column 4 shows the account balance at the end of the year, not including the mortality gains that would result from the forfeitures from other members who died that year. Column 5 shows the worker’s probability of dying during that year. Finally, Column 6 shows the closing balance in the worker’s account including the mortality gains that result from the forfeitures from other members who died that year. The final row of Table 8 shows that a worker who lived (and worked) from age 35 through age 64 and retired at 65 would have a final pre-retirement salary of $155,933 (Column 2) and would have a starting retirement balance in her tontine pension account of $843,376 (Column 6).

201 It is calculated as the sum of the prior year’s balance multiplied by (1 plus the interest rate) plus the current year’s contribution multiplied by the square root of (1 plus the interest rate).

202 This is the expected value of the balance that results from mortality gains. See supra note 198. It is computed by taking the preliminary balance in Column 4 and dividing it by (1 minus the death probability) in Column 5. For example, the closing balance in the account of an employee at age 64 is $843,377 ($843,376.82 = $833,161/(1 - 0.012113)) (minor error due to rounding).
Table 8: Calculation of the Retirement Balance

<table>
<thead>
<tr>
<th>Age</th>
<th>Salary</th>
<th>Contribution</th>
<th>Preliminary Balance</th>
<th>Death Probability</th>
<th>Closing Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>$50,000</td>
<td>$5000</td>
<td>$5172</td>
<td>0.001261</td>
<td>$5179</td>
</tr>
<tr>
<td>36</td>
<td>$52,000</td>
<td>$5200</td>
<td>$10,920</td>
<td>0.001332</td>
<td>$10,935</td>
</tr>
<tr>
<td>37</td>
<td>$54,080</td>
<td>$5408</td>
<td>$17,294</td>
<td>0.001420</td>
<td>$17,319</td>
</tr>
<tr>
<td>38</td>
<td>$56,243</td>
<td>$5624</td>
<td>$24,349</td>
<td>0.001527</td>
<td>$24,386</td>
</tr>
<tr>
<td>39</td>
<td>$58,493</td>
<td>$5849</td>
<td>$32,144</td>
<td>0.001653</td>
<td>$32,197</td>
</tr>
<tr>
<td>40</td>
<td>$60,833</td>
<td>$6083</td>
<td>$40,743</td>
<td>0.001796</td>
<td>$40,816</td>
</tr>
<tr>
<td>41</td>
<td>$63,266</td>
<td>$6327</td>
<td>$50,218</td>
<td>0.001955</td>
<td>$50,316</td>
</tr>
<tr>
<td>42</td>
<td>$65,797</td>
<td>$6580</td>
<td>$60,644</td>
<td>0.002133</td>
<td>$60,774</td>
</tr>
<tr>
<td>43</td>
<td>$68,428</td>
<td>$6843</td>
<td>$72,107</td>
<td>0.002332</td>
<td>$72,275</td>
</tr>
<tr>
<td>44</td>
<td>$71,166</td>
<td>$7117</td>
<td>$84,696</td>
<td>0.002550</td>
<td>$84,912</td>
</tr>
<tr>
<td>45</td>
<td>$74,012</td>
<td>$7401</td>
<td>$98,512</td>
<td>0.002786</td>
<td>$98,787</td>
</tr>
<tr>
<td>46</td>
<td>$76,973</td>
<td>$7697</td>
<td>$113,665</td>
<td>0.003041</td>
<td>$114,011</td>
</tr>
<tr>
<td>47</td>
<td>$80,052</td>
<td>$8005</td>
<td>$130,273</td>
<td>0.003322</td>
<td>$130,707</td>
</tr>
<tr>
<td>48</td>
<td>$83,254</td>
<td>$8325</td>
<td>$148,468</td>
<td>0.003630</td>
<td>$149,009</td>
</tr>
<tr>
<td>49</td>
<td>$86,584</td>
<td>$8658</td>
<td>$168,396</td>
<td>0.003963</td>
<td>$169,066</td>
</tr>
<tr>
<td>50</td>
<td>$90,047</td>
<td>$9005</td>
<td>$190,215</td>
<td>0.004326</td>
<td>$191,042</td>
</tr>
<tr>
<td>51</td>
<td>$93,649</td>
<td>$9365</td>
<td>$214,102</td>
<td>0.004707</td>
<td>$215,114</td>
</tr>
<tr>
<td>52</td>
<td>$97,395</td>
<td>$9740</td>
<td>$240,247</td>
<td>0.005086</td>
<td>$241,475</td>
</tr>
<tr>
<td>53</td>
<td>$101,291</td>
<td>$10,129</td>
<td>$268,856</td>
<td>0.005455</td>
<td>$270,333</td>
</tr>
<tr>
<td>54</td>
<td>$105,342</td>
<td>$10,534</td>
<td>$300,150</td>
<td>0.005827</td>
<td>$301,910</td>
</tr>
<tr>
<td>55</td>
<td>$109,556</td>
<td>$10,956</td>
<td>$334,376</td>
<td>0.006234</td>
<td>$336,473</td>
</tr>
<tr>
<td>56</td>
<td>$113,938</td>
<td>$11,394</td>
<td>$371,812</td>
<td>0.006685</td>
<td>$374,315</td>
</tr>
<tr>
<td>57</td>
<td>$118,496</td>
<td>$11,850</td>
<td>$412,774</td>
<td>0.007166</td>
<td>$415,753</td>
</tr>
<tr>
<td>58</td>
<td>$123,236</td>
<td>$12,234</td>
<td>$457,604</td>
<td>0.007677</td>
<td>$461,144</td>
</tr>
<tr>
<td>59</td>
<td>$128,165</td>
<td>$12,817</td>
<td>$506,681</td>
<td>0.008233</td>
<td>$510,888</td>
</tr>
<tr>
<td>60</td>
<td>$133,292</td>
<td>$13,292</td>
<td>$560,438</td>
<td>0.008854</td>
<td>$565,444</td>
</tr>
<tr>
<td>61</td>
<td>$138,623</td>
<td>$13,862</td>
<td>$619,364</td>
<td>0.009552</td>
<td>$625,338</td>
</tr>
<tr>
<td>62</td>
<td>$144,168</td>
<td>$14,417</td>
<td>$684,024</td>
<td>0.010233</td>
<td>$691,159</td>
</tr>
<tr>
<td>63</td>
<td>$149,935</td>
<td>$14,994</td>
<td>$755,050</td>
<td>0.011172</td>
<td>$763,580</td>
</tr>
<tr>
<td>64</td>
<td>$155,933</td>
<td>$15,593</td>
<td>$833,161</td>
<td>0.012113</td>
<td>$843,376</td>
</tr>
</tbody>
</table>
C. Calculation of the Monthly Tontine-Pension Distributions

At retirement, the expected monthly payout is identical to the actual monthly payout of an actuarially fair annuity. As we have seen, the monthly payout of an actuarially fair annuity equals the account balance divided by the applicable monthly annuity factor. For example, consider a worker who worked from age 35 through age 64 and retired on the last day of that year. We can see from the last entry in Table 8 that the closing account balance for that worker was $843,376. Assuming that she wants to draw level monthly tontine pension payments for the rest of her life, she should start by looking at Column 5 of Appendix Table 1, which shows that the uniform monthly annuity factor for the first month after she turns 65 is almost 118. Therefore, the first monthly distribution for a uniform tontine pension would be $7166 ($7165.84 = $843,376/117.6939).

Alternatively, if this retiree instead wanted inflation-adjusted payments for the rest of her life, Column 6 of Appendix Table 1 shows that the initial monthly annuity factor for the first month after she turns 65 is almost 152. Accordingly, the first monthly distribution for an inflation-adjusted tontine pension would be just $5549 ($5548.98 = $843,376/151.9876).

Figure 2 plots the expected payouts from these uniform and inflation-adjusted tontine pensions over time. The plot is for a member retiring on her 65th birthday. The uniform payout is the amount of the monthly payment in dollars. Ideally it is a constant $7166 per month for life—and that is what an actuarially fair life annuity would pay. The actual payments would fluctuate a little bit around that value, but as the plot shows, the uniform payout curve is relatively smooth. Of course, that is what we would expect given that our model assumes a constant 7% rate of return and a constant 3% inflation rate. Consequently, monthly fluctuations result only from the randomness of deaths in the population, but with approximately 70,000 retirees at any point in time, those fluctuations are insignificant.

---

203 See supra notes 140-141 and accompanying text.
204 For comparison, an annuity purchased from a commercial insurer would make a fixed monthly payment but of a lower amount depending on the insurer’s load charge. For a typical load of 10%, the monthly payment would fall to just $6449.40 ($6449.40 = $7166 × 90%).
By contrast, the inflation-adjusted payout starts at $5549 per month and increases at an annual rate of 3% per year—that is what an actuarially fair life annuity with a 3% escalator would pay (and the model assumes a constant 3% inflation rate). Again, the actual payments will fluctuate a little bit around those values, but as the plot shows, the inflation-adjusted payout curve is also quite smooth.

D. Adequacy

All in all, we have shown how a large employer could use a tontine pension to provide retirement benefits for its employees. Given the assumptions in our model, Table 8 showed that our hypothetical retiree would have a final salary of $155,933 at age 64 and would have accumulated $843,376 by age 65. The latter sum would support a uniform tontine pension
of around $7166 per month for life or an inflation-adjusted tontine pension that starts at around $5549 per month at age 65 and increases in later months.

It is relatively easy to determine how much pre-retirement income this 30-year, 10%-of-salary tontine pension would replace. For example, multiplying the uniform monthly benefit of $7166 by 12 months yields an annual tontine pension of $85,992 ($85,992 = 12 × $7166), and it is easy to see that the tontine pension would replace 55.1% of pre-retirement earnings in the first year of retirement (i.e., a "replacement ratio" of 55.1% (0.5514676 = $85,992/$155,933)).205 Similarly, the inflation-adjusted monthly benefit would yield an annual tontine pension starting at around $66,588 ($66,588 = 12 × $5549) and a replacement ratio of around 42.7% of pre-retirement earnings (0.4270295 = $66,588/$155,933).206 In addition to these tontine pensions, however, our retiree would almost certainly receive Social Security benefits, and those Social Security benefits would replace another 35% to 40% of her pre-retirement income.207

All in all, it seems that a 10%-of-salary tontine pension would generate a pretty substantial retirement benefit for the typical worker. Moreover, raising the tontine pension contribution rate (e.g., above 10%) or increasing

---

205 The replacement ratio is the ratio of income in retirement to income pre-retirement. The desired replacement ratio is almost always assumed to be less than 100% because of the elimination of work-related expenses, because some pre-retirement income was devoted to saving for retirement, and because Social Security benefits are taxed more favorably than earned income. See AON CONSULTING, REPLACEMENT RATIO STUDY: A MEASUREMENT TOOL FOR RETIREMENT PLANNING 24 (2008), available at http://www.aon.com/about-aon/intellectual-capital/attachments/human-capital-consulting/RRStudy070308.pdf (estimating that required replacement ratios ranged from 77% for a person earning $80,000 a year in 2008 to 94% for a person earning $20,000 that year).

206 Because of the impact of the 3% inflation assumption and the passage of time on the monthly tontine pension annuity factors, our retiree could expect that her monthly tontine pension benefits for the 11 months following her initial month of retirement would be slightly larger than the $5549 that she would receive in the first month of that retirement year. Accordingly, she should receive an annual pension of slightly more than $66,588 at age 65 and have a replacement ratio of slightly higher than 42.7%.

the number of working years (e.g., above 30) covered by the tontine pension would result in retirees receiving even more benefits and having even higher replacement ratios.

E. Tontine Pensions in the Real World

Our model does a respectable job of showing how a tontine pension could work in the real world. To be sure, the assumptions of the model are somewhat rigid. In the real world, inflation is not always 3% per year, wages do not always increase by 4% per year, and investments do not always earn a 7% rate of return. Each of those parameters is highly variable, although their average values are probably pretty close to our assumed values. In general, that real world variability could easily result in retirees receiving smaller (or larger) monthly distributions from their tontine pensions. To the extent that that real-world volatility puts retirement income security at risk, it is worth reiterating that either raising the tontine pension contribution rate or increasing the number of working years covered by the tontine pension would result in retirees receiving more benefits and having higher replacement ratios.

IV. REPLACING THE CALIFORNIA STATE TEACHERS’ RETIREMENT SYSTEM WITH A TONTINE PENSION

In this Part, we consider how a tontine pension for a large employer would work. Given the strictures of ERISA and federal securities regulation laws, we acknowledge that it may be a challenge for a private pension plan sponsor to create a tontine pension under current law.208 On the other hand, public employers are exempt from most of ERISA’s pension regulations.209 Accordingly, we believe that a state government could easily create a tontine pension that would not run afoul of federal law. As we have seen, such a tontine pension would be fully funded and would make annuity-like payments to retirees for as long as they lived.210

As most states already have pension plans that cover most of their employees, what we are really talking about here is the prospect of replacing

208 For a more thorough discussion of the legal issues involving tontine pensions, see infra Section V.B.
210 See supra Section II.D. We recognize that many governments use their pension plans to provide disability benefits, and some also use their pension plans to provide retiree health benefits. However, for simplicity we have ignored both disability benefits and retiree health benefits in this Article.
an existing state pension plan with a tontine pension. In particular, some states might want to replace their underfunded traditional defined benefit pension plans with tontine pensions. For our example, this Part considers whether California might want to replace the $74 billion underfunded California State Teachers’ Retirement System (CalSTRS) defined benefit plan with a tontine pension. 211

A. Background on the California State Teachers’ Retirement System

CalSTRS is the largest educator-only pension in the world, with a membership of 868,493 and assets of approximately $187.1 billion as of October 31, 2014. 212 One of the largest programs that CalSTRS administers is its traditional defined benefit retirement plan, where benefits are based on a member’s years of service, age, and highest compensation. 213 Essentially, members receive an annual retirement benefit \( B \) that is equal to 2% multiplied by the number of years of service \( yos \) multiplied by final average compensation \( fac \) \( (B = 2\% \times yos \times fac) \).

For the fiscal year that ended on June 30, 2013, the CalSTRS traditional defined benefit pension had 416,643 active members with an average annual salary of $61,153 and 269,274 retired members and beneficiaries with an average annual retirement benefit of $43,308. 214 Also, as of June 30, 2013, the CalSTRS defined benefit plan was only 66.9% funded, with an unfunded liability of almost $74 billion. 215 The normal retirement benefit cost, expressed as a percentage of total compensation, was 16.818%. 216 In addition, as of June 30, 2013, CalSTRS needed another 14.620% of total compensation to amortize its $74 billion unfunded liability over 30 years. 217

211 See MILLIMAN, supra note 19, at 10.
214 MILLIMAN, supra note 19, at 10.
215 Id.
216 Id. at 18 tbl.1. Under the entry-age normal cost accounting method, the normal cost is calculated to produce a level cost over each employee’s career (i.e., a level percentage of payroll). The normal cost generally represents the expected cost of projected benefits attributable to work performed and pension benefits earned in the current plan year. Id. at 15.
217 Id. at 47 tbl.15.
B. Replacing the California State Teachers’ Retirement System Defined Benefit Plan with a Tontine Pension

There are a variety of possible ways to replace a traditional pension like the CalSTRS defined benefit plan with a tontine pension. Perhaps the most likely approach would be to keep the current defined benefit plan for all current employees but to close entry to that plan and require all new employees to join a newly created tontine pension.218

A more interesting approach would be for CalSTRS to freeze its current defined benefit plan and add a new tontine pension for all future benefit accruals.219 At retirement, beneficiaries would then receive the defined benefit plan benefits that they have already accrued, but they would not accrue any additional benefits under their traditional defined benefit plan; instead, future contributions would be made to a new tontine pension. Theoretically, CalSTRS would freeze its defined benefit plan and add a tontine pension with future retirement contributions set at, for example, 16.818% of compensation (i.e., the current CalSTRS defined benefit plan’s normal cost rate).220 Going forward, such a plan would be roughly as generous as the current plan, but CalSTRS would never again have to worry about underfunding as a result of future benefit accruals. To be sure, this way of replacing the CalSTRS defined benefit plan with a tontine pension would do nothing to reduce its $74 billion unfunded liability, and that obligation would still need to be met by the state of California.

We do not mean to suggest that replacing the CalSTRS defined benefit plan with a tontine pension would be politically easy. We merely suggest that a tontine pension could provide an alternative way of providing lifetime retirement income to California teachers, and we reiterate that unlike traditional defined benefit plans—which are often underfunded—a tontine pension can never become underfunded.

---

218 Cf. Jonathan Barry Forman, Public Pensions: Choosing Between Defined Benefit and Defined Contribution Plans, 1999 MICH. ST. L. REV. 187, 208-10 (discussing various ways to transition from a traditional defined benefit plan to a defined contribution plan, but noting the difficulties inherent in making this switch).
219 See id. at 210 (describing this approach).
220 See supra note 216 and accompanying text.
V. SOLVING THE TECHNICAL PROBLEMS OF CREATING A TONTINE PENSION

Finally, this Part addresses some of the technical issues raised by tontine pensions.

A. Taxation of Benefits

Presumably, tontine pension benefits would be taxed like other pension benefits.\textsuperscript{221} Employer contributions to a tontine pension should be excluded from the income of employees; the tontine pension fund’s earnings should be exempt from tax; and retirees should be taxed only when they receive their monthly tontine-pension distributions. At the same time, the employer should be allowed a current deduction for its contributions to the tontine pension.\textsuperscript{222} We note that the prospectus for CREF suggests that CREF’s tontine-like pensions and annuities are taxed in accordance with these principles.\textsuperscript{223}

B. Legal Issues

Although not a certainty, it appears that tontine funds, tontine annuities, and tontine pensions are all legal. As previously mentioned, investigations of the insurance industry in New York led to the enactment of legislation in 1906 that all but banned tontines.\textsuperscript{224} To be sure, the legislation did not specifically prohibit the sale of tontines; instead, it just made it difficult for companies to defer payments beyond one year.\textsuperscript{225} Many states followed New York’s lead, and tontines soon fell out of favor.\textsuperscript{226}

Much has changed since the beginning of the twentieth century, however. In particular, financial products today do a much better job at recordkeeping,\textsuperscript{227} and investment assets are usually held by independent

\textsuperscript{221} See supra subsection I.B.1.
\textsuperscript{222} To the extent that any employees make (or are deemed to make) any after-tax contributions to their tontine pension funds, they should be allowed to recover those contributions tax-free, just as they could with a typical pension or annuity. See supra note 33.
\textsuperscript{223} See TIAA–CREF FIN. SERVS., supra note 156, at 81-87 (describing the tax implications of similar existing pension plans).
\textsuperscript{224} See supra note 14 and accompanying text.
\textsuperscript{225} COOPER, supra note 9, at 56.
\textsuperscript{226} Id. at 57.
\textsuperscript{227} Today, for example, there are numerous laws that govern the securities industry. The Laws that Govern the Securities Industry, U.S. SEC. & EXCHANGE COMMISSION, http://www.sec.gov/about/laws.shtml (last visited Jan. 16, 2015), archived at http://perma.cc/EWG5-9VWW. Also, we have seen that ERISA imposes a number of recordkeeping and reporting requirements on pension plan sponsors. See supra subsection I.B.3.
custodians. Also, most states have softened their views on lotteries and gambling. Accordingly, there should be less suspicion about tontine financial products. In fact, today, only Louisiana and South Carolina have statutes that actually ban tontines. All in all, it seems likely that tontine financial products could be designed in ways that would survive state regulatory scrutiny. Indeed, as we have seen, CREF is arguably a tontine, and it operates in, and is expressly regulated, by the State of New York, as well as by the insurance regulators of certain other states. Any state that wished to set up a tontine pension for its own workers could enact a statute to permit that state to do so.

Tontine financial products should also be able to withstand federal regulatory scrutiny. As long as tontine financial products maintain good records, make adequate disclosures, and ensure that the underlying investment assets are held by independent custodians, the SEC should be satisfied.

For some tontine pensions, ERISA may present some regulatory hurdles. However, unless they are “established or maintained” by an employers or a union, tontine funds and tontine annuities would not be “employee benefit plans” within the meaning of ERISA’s section 4 coverage rule, and therefore would not be subject to ERISA.

230 McKeever, supra note 9, at 514.
231 See supra notes 153-160 and accompanying text.
232 TIAA–CREF FIN. SERVS., supra note 159, at B-44.

Moreover, to the extent that any tontine annuities might be subject to ERISA, we believe that ERISA’s insurance savings clause is relevant with respect to any tontine annuity viewed as an insurance product under the applicable state’s law. In that regard, ERISA’s preemption clause provides that ERISA “shall supersede any and all State laws . . . [that] relate to any employee benefit plan”; however, the savings clause then exempts from preemption any state law “which regulates insurance, banking, or securities.” Id. § 514(a), (b)(2)(A), 29 U.S.C. §§ 1144(a), (b)(2)(A). Congress generally left the regulation of insurance products to the states. Presumably, tontine annuities sold by insurance companies would be subject to regulation by state insurance regulators. But what about tontine annuities sold by a discount broker? Are these just investment products or
On the other hand, tontine pensions established by employers or unions would be “employee benefit plans” within the meaning of ERISA.\textsuperscript{234} As mentioned above, government plans are exempt from ERISA, so state and local governments could set up tontine pensions for their employees without having to comply with ERISA.\textsuperscript{235}

Conversely, private-sector tontine pension plans would be subject to ERISA. The next question is whether there are any provisions of ERISA that would prevent private employers from creating tontine pensions for their employees. To be sure, traditional pensions exhibit tontine characteristics; for example, those who live longer will accrue more (monthly) benefits than those who die younger.\textsuperscript{236}

Nevertheless, several provisions of ERISA may pose regulatory challenges for private-sector tontine pensions.

For example, with respect to defined benefit plans, Internal Revenue Code section 401(a)(8) indicates that “forfeitures must not be applied to increase the benefits any employee would otherwise receive under the plan.”\textsuperscript{237} With a tontine pension, all participants are entitled to a benefit that approximates an actuarially fair annuity. Therefore, those who live longer will get more (monthly) benefits than those who die younger. Because this is exactly what happens under a traditional defined benefit plan, we believe that tontine pensions should not be viewed as applying forfeitures to increase the benefits of other employees in violation of section 401(a)(8), and accordingly, we believe that the Internal Revenue Service should be willing to issue guidance to that effect (e.g., a private letter ruling). Moreover, we note that defined benefit plans have always been allowed to invest in annuities for their employees. Accordingly, we believe that defined benefit plans would be permitted to invest in tontine annuities. Of course, employers might prefer to operate their tontine pensions on a fully funded defined contribution plan platform. In that case, section 401(a)(8) would not be applicable.

ERISA’s vesting rules may also pose a regulatory challenge for tontine pensions. For example, could a tontine pension meet the three-year cliff

\textsuperscript{234} In general, a tontine pension would be an “employee benefit plan . . . . established or maintained by” an employer or employee organization within the meaning of ERISA section 4. Id. § 3(3), 29 U.S.C. § 1002(3) (2012); id. § 4, 29 U.S.C. § 1003(a) (2012).


\textsuperscript{236} COOPER, supra note 9, at 61 (explaining the distribution of benefits over time in traditional pensions).

\textsuperscript{237} I.R.C. § 401(a)(8) (2012).
vesting rule that generally applies to employer contributions?\textsuperscript{238} How do we interpret the fact that a single worker with a tontine pension account would lose everything in her account at death, even if she had worked for the employer for more than three years? Is forfeiture at death allowed in a defined contribution plan investment?

One approach is to ask whether an employer with a defined contribution plan could use employer contributions each year to buy commercial life annuities for each employee. We believe an employer could do so. Because tontine annuities would work just like commercial annuities, an employer should be able to design a defined contribution plan that invests in tontine annuities for its employees, even if those tontine annuities become worthless at death.\textsuperscript{239}

ERISA’s fiduciary obligation rules could also pose some regulatory challenges for tontine pensions.\textsuperscript{240} For example, pension plans must be operated for the exclusive benefit of employees or their beneficiaries, and plan fiduciaries must act prudently and diversify the plan’s investments.\textsuperscript{241} Again, we see no reason to be concerned about a pension operating as a tontine


\textsuperscript{239} Another option is to begin by considering an individual with an IRA. IRAs are not subject to ERISA, but the Internal Revenue Code rules that govern IRAs are very similar to the ERISA rules governing defined contribution plans. For example, both IRAs and pensions receive favorable tax treatment, and both are subject to the prohibited transactions rules. See supra Section I.B. We do not believe that there is anything in the Internal Revenue Code that would prevent an individual from having her IRA invest in a tontine fund or in a tontine annuity. Nor do we think that ERISA would prevent a participant with a self-directed 401(k) plan from investing in a tontine fund or annuity.

Finally, there is no doubt that an employer can create a defined contribution plan, make contributions to that plan on behalf of its employees, and invest those contributions for the benefit of its employees. The question comes down to whether a plan sponsor can invest employer contributions in a tontine fund or tontine annuity knowing, as we do, that each employee will lose the balance in her account when she dies. We see no reason why a plan sponsor would be prohibited from doing so. (Granted, the spousal protection rules might impose forfeiture limits with respect to married participants. We discuss those rules infra subsection V.D.3).


pension or investing in tontine annuities, and we believe that the government would issue guidance supporting our position.\footnote{Cf. Selection of Annuity Providers for Individual Account Plans, 72 Fed. Reg. 52,021 (proposed Sept. 12, 2007) (to be codified at 29 C.F.R. pt. 2550) (proposing the establishment of a “safe harbor” for selecting annuity providers to distribute benefits from “individual account plans covered by title I” of ERISA).}

We believe that tontine funds, tontine annuities, and tontine pensions could be designed in ways that comply with applicable state and federal laws.

C. Dealing with Market Volatility

Unlike a traditional defined benefit pension plan that makes fixed or inflation-adjusted benefit payments, tontine pension benefit payments would be volatile. Monthly tontine-pension distributions would vary with fluctuations in the value of the underlying assets and with the variability inherent in the indeterminate timing of the deaths of the other participants in the tontine pension. The fluctuations attributable to the randomness of the deaths of other participants would largely disappear as long as there are enough participants in the tontine pension.\footnote{See supra subsection II.B.3.a.}

In contrast, the volatility due to fluctuations in the value of the underlying assets will not disappear. This is the same problem that any investor with a defined contribution plan or variable annuity confronts.\footnote{See supra notes 136-138 and accompanying text.} For example, an investor who used the 4\% rule to withdraw $40,000 from her individual account in 2007 when her stock portfolio was worth $1,000,000 could only withdraw around $20,000 in 2009 when that portfolio was worth just $500,000. An investor can minimize the effects of market volatility by investing conservatively in bonds, but the expected earnings on her portfolio could fall dramatically.\footnote{According to one projection, over the next 10 years, the expected return on U.S. stocks will be 7.25\%, while the expected return on U.S. Treasury bonds will be just 0.50\%. See BNY MELLON, 10-YEAR CAPITAL MARKET RETURN ASSUMPTIONS: CALENDAR YEAR 2013 (2013), available at http://us.bnymellonam.com/core/library/documents/knowledge/market_commentary/bny_mellon_10_year_capital_market_return_assumptions_2013.pdf (presenting 10-year capital market return assumptions based on social and economic changes).}

Of course, planning for that market volatility can help mitigate its impact. Wise consultants with irregular earnings generally spend no more money in the months that they get commissions than they do in the months that they do not. Similarly, the investor discussed in the previous paragraph could have spent just $30,000 of the $40,000 she withdrew in 2007 and saved the other $10,000 to spend in 2009 when she withdrew just $20,000. That is,
individuals can smooth their consumption by underspending in the good years so that they can spend more in the lean years. Smoothing products, even “smoothed income annuities” can be purchased in the marketplace.\textsuperscript{246}

A tontine pension could itself be designed to provide smoother distributions. For example, monthly distributions could be smoothed over a one-year or even a five-year period.\textsuperscript{247} When the tontine pension administrator determined that a certain monthly distribution would be higher than the average distribution over the prior five years, the distribution could be split. A basic distribution could go to the participant’s bank account immediately, and the excess could go into a “holding account” for the participant. In a later month when the tontine pension administrator determined that the distribution would otherwise be lower than the average for the prior five years, the holding account could be tapped to provide a larger distribution. The funds in the holding account could be invested with all of the other assets held by the tontine pension, and presumably, at that member’s death, any balance in her holding account could be paid to her estate.

In short, income smoothing could be accomplished either inside or outside of a tontine pension. In any event, the volatility in monthly distributions attributable to fluctuations in the value of the underlying investment assets held in a tontine pension is no worse a problem for tontine pensions than it is for defined contribution plans or variable annuities.

D. Gender Issues

1. In General

While insurance companies can typically price the annuities that they offer to men and women differently, pension plans cannot offer different pricing based on gender.\textsuperscript{248} Pension plans cannot require higher contributions from women or pay women lower benefits.\textsuperscript{249} Therefore,

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{248}See supra note 72 and accompanying text (explaining that Title VII of the Civil Rights Act prohibits pension plans from requiring higher contributions from women than men or paying women lower benefits than men).
\item \textsuperscript{249}Id.
\end{itemize}
\end{footnotesize}
when an employee retires with a traditional defined benefit pension, the retiree will see the same monthly pension benefits for life, regardless of gender. For example, CalSTRS pays identical pensions to retired men and women teachers who have the same service records.\textsuperscript{250} To be sure, defined benefit plan actuaries take the gender of participants and their partners into account when determining the contributions that the plan sponsor needs to make. Retiring women can expect to collect more monthly benefit checks than their male counterparts, but the monthly payments must be equal for men and women.\textsuperscript{251}

Tontine funds and tontine annuities could account for gender.\textsuperscript{252} However, a tontine pension, like a traditional pension, would not be permitted to discriminate based on gender because Title VII of the Civil Rights Act of 1964 forbids this type of discrimination.\textsuperscript{253} A tontine pension can comply with this gender neutrality requirement by using unisex life expectancy tables, as this Article does with its model tontine pension.\textsuperscript{254}

2. Employee Contributions

Title VII’s gender-neutrality requirement somewhat undermines the attractiveness of allowing participants to make additional voluntary contributions to their employer-provided tontine pensions. To be sure, allowing employees to make supplemental contributions to their tontine pensions would enhance employees’ retirement incomes, just as voluntary contributions to 401(k) plans increase participants’ nest eggs and their retirement income. However, tontine pensions would be a better investment

---

\textsuperscript{250} See supra Section IV.A (providing background on CalSTRS).

\textsuperscript{251} On the other hand, a defined contribution plan can distribute lump sums to its retirees with the knowledge that the commercial annuities available to the retirees from private insurers will differ based on gender. As noted above, a 65-year-old man who purchased a $100,000 annuity in January of 2014 could receive $6864 a year for life, while a 65-year-old woman would receive $6408 a year because of her longer life expectancy. See supra subsection I.C.2 (explaining lifetime annuities). But see Heen, supra note 121 (discussing why we should ban gender discrimination in the sale of commercial annuities).

\textsuperscript{252} See supra Sections II.B-C (providing an overview of tontine funds and tontine annuities).

\textsuperscript{253} See supra note 72 and accompanying text (explaining that Title VII of the Civil Rights Act prohibits pension plans from requiring higher contributions from women than men or paying women lower benefits than men); see also Spirt v. Teachers Ins. & Annuity Ass’n, 691 F.2d 1054, 1066 (2d Cir. 1982) (finding that defendant’s use of sex-distinct tables for calculating contributions to a pension plan constituted unequal treatment on the basis of sex), vacated on other grounds, 463 U.S. 1223 (1983).

\textsuperscript{254} See supra note 200 and accompanying text. Unisex tables are not a perfect solution, because they are less accurate than gender-specific tables. Unisex tables would, however, ensure that same-age men and women who make identical contributions receive identical monthly distributions, which is what Title VII requires for pensions.
for women than men, given their relative life expectancies. The typical man would be better off investing in a 401(k) plan or IRA (where gender is irrelevant), or in a typical commercial annuity sold by an insurance company (where gender can be considered).  

3. Qualified Joint and Survivor Annuities & Qualified Domestic Relations Orders

Under ERISA, defined benefit plans (and some defined contribution plans) are required to provide a qualified joint-and-survivor annuity (QJSA) as the normal benefit payment for married participants, unless the spouse consents to another form of distribution. These plans are also required to provide a qualified pre-retirement survivor annuity (QPSA) option in case the worker dies before retirement. ERISA-covered pension plans also allow state courts to divide the pension benefits of married couples through qualified domestic relations orders (QDROs). Although not covered by ERISA, many public pension plans provide similar spousal protections.

Tontine pensions could also be designed to provide spousal protections. First, with respect to survivor benefits, rather than having a married participant forfeit her entire account balance at her death, a tontine pension could provide QJSAs and QPSAs. For example, when a participant dies, she might forfeit half of the balance in her account; the remaining half could be

---

255 Again, see Heen, supra note 121 for a discussion as to why we should ban gender discrimination in the sale of annuities.

256 I.R.C. § 401(a)(11) (2012) (“[A] trust forming part of such plan shall not constitute a qualified trust under this section unless . . . the accrued benefit payable to such participant is provided in the form of a qualified joint and survivor annuity.”); id. § 417(a) (permitting participants to elect to waive the qualified joint and survivor annuity form, but requiring the participant’s spouse to consent in writing); Employee Retirement Income Security Act of 1974 § 205(a)-(c), 29 U.S.C. § 1055(a)-(c) (2012) (same). A QJSA is an immediate annuity for the life of the pension plan participant and a survivor annuity for the life of the participant’s spouse. Id. § 205(d)(1), 29 U.S.C. § 1055(d)(1). The amount of the survivor annuity may not be less than 50% nor more than 100% of the amount payable during the time the participant and spouse are both alive. Id., 29 U.S.C. § 1055(d)(1).

257 Id. § 205(a), 29 U.S.C. § 1055(a). A QPSA typically pays an annuity that is equal to the survivor’s portion of the QJSA. Id. § 205(e), 29 U.S.C. § 1055(e).


retitled in the name of the surviving spouse. Second, a tontine pension could allow divorcing spouses to secure domestic relations orders that transfer a portion of the participant spouse’s tontine pension to the other spouse. This could allow the transferred portion to be retitled in the name of the transferee spouse.

CONCLUSION

In this Article, we showed how large employers could use tontine pensions to provide retirement income for their employees. We developed a model tontine pension and used that model to show the retirement benefits that a typical worker could earn with a 10%-of-salary tontine pension. Over the course of a 30-year career, we estimated that a typical retiree would earn a uniform tontine pension that would initially replace approximately 55% of her pre-retirement earnings. Alternatively, that retiree would earn an inflation-adjusted tontine pension that would replace approximately 43% of her pre-retirement earnings.

These tontine pensions have two major advantages over traditional defined benefit plan pensions. First, unlike traditional pensions, which are frequently underfunded, tontine pensions would always be fully funded. Second, unlike traditional pensions, where the plan sponsor must bear all the investment and actuarial risks, with a tontine pension, the plan sponsor would bear neither of those risks. These two features make the tontine pension a particularly attractive alternative for employers who care about providing retirement income security for their employees but want to avoid the risks associated with having a traditional pension.

Tontine pensions also offer a possible solution to the chronic underfunding of state and local pension plans. For example, we showed how California could replace its $74 billion underfunded CalSTRS defined benefit plan with a tontine pension and never again have to worry about underfunding attributable to future benefit accruals.

Finally, a tontine pension would closely resemble an actuarially fair variable life annuity, but could be run by a low-fee discount broker. No money would need to be set aside for insurance agent commissions or for insurance

---

260 The tontine pension of a married couple might be shared between the spouses along the lines of earnings sharing. See, e.g., FORMAN, supra note 23, at 205-06 (discussing the possibility of earnings sharing for Social Security).

261 QDROs can present adverse selection and moral hazard issues. For example, what, if anything, should be done to prevent a dying spouse from getting a divorce and using a QDRO to transfer her tontine pension to her ex-spouse, rather than forfeiting it to the surviving members in her tontine pension plan?
company reserves, risk-taking, and profits. This means that tontine pensions would provide significantly higher benefits to retirees than commercial annuities.
Appendix Table 1 is based on the Social Security Administration's 2009 unisex life table.\textsuperscript{262} For individuals aged 35 through 119, Column 1 shows their age ($x_i$), Column 2 shows their life expectancy ($e_i$), and Column 3 shows their death probability ($q_i$). Column 4 shows the force-of-mortality probabilities that we derived,\textsuperscript{263} and Columns 5 and 6 show the uniform and inflation-adjusted monthly annuity factors that we derived for the first month of each year starting with age 65.\textsuperscript{264}

### Appendix Table 1: Unisex Life Tables, 2009, with Force-of-Mortality Probabilities, and Monthly Annuity Factors\textsuperscript{265}

<table>
<thead>
<tr>
<th>Age ($x_i$)</th>
<th>Life Expectancy (years) ($e_i$)</th>
<th>Death Probability ($q_i$)</th>
<th>Force-of-Mortality Probability ($f_i$)</th>
<th>Uniform Monthly Annuity Factors for the First Month of the Year</th>
<th>Inflation-adjusted Monthly Annuity Factors for the First Month of the Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>44.90</td>
<td>0.001261</td>
<td>0.001262</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>36</td>
<td>43.95</td>
<td>0.001332</td>
<td>0.001333</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>37</td>
<td>43.01</td>
<td>0.001420</td>
<td>0.001421</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>38</td>
<td>42.07</td>
<td>0.001527</td>
<td>0.001528</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>39</td>
<td>41.14</td>
<td>0.001653</td>
<td>0.001655</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>40</td>
<td>40.20</td>
<td>0.001796</td>
<td>0.001798</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>41</td>
<td>39.27</td>
<td>0.001955</td>
<td>0.001957</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>42</td>
<td>38.35</td>
<td>0.002133</td>
<td>0.002135</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>43</td>
<td>37.43</td>
<td>0.002332</td>
<td>0.002334</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>44</td>
<td>36.52</td>
<td>0.002550</td>
<td>0.002553</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

\textsuperscript{262} Bye, supra note 109.

\textsuperscript{263} See supra note 111 and accompanying text.

\textsuperscript{264} See supra notes 141 & 143 and accompanying text.

\textsuperscript{265} This data is derived from Bye, supra note 109, and authors' computations. The monthly annuity factors were determined using an interest rate of 7% and an inflation rate of 3%.
<table>
<thead>
<tr>
<th>Age (x_i)</th>
<th>Life Expectancy (years) (e_i)</th>
<th>Death Probability (q_i)</th>
<th>Force-of-Mortality Probability (f_i)</th>
<th>Uniform Monthly Annuity Factors for the First Month of the Year</th>
<th>Inflation-adjusted Monthly Annuity Factors for the First Month of the Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>35.61</td>
<td>0.002786</td>
<td>0.002790</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>46</td>
<td>34.71</td>
<td>0.003041</td>
<td>0.003046</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>47</td>
<td>33.81</td>
<td>0.003322</td>
<td>0.003328</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>48</td>
<td>32.92</td>
<td>0.003630</td>
<td>0.003637</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>49</td>
<td>32.04</td>
<td>0.003963</td>
<td>0.003971</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>50</td>
<td>31.17</td>
<td>0.004326</td>
<td>0.004336</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>51</td>
<td>30.30</td>
<td>0.004707</td>
<td>0.004718</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>52</td>
<td>29.44</td>
<td>0.005086</td>
<td>0.005099</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>53</td>
<td>28.59</td>
<td>0.005455</td>
<td>0.005470</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>54</td>
<td>27.74</td>
<td>0.005827</td>
<td>0.005844</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>55</td>
<td>26.90</td>
<td>0.006234</td>
<td>0.006253</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>56</td>
<td>26.07</td>
<td>0.006685</td>
<td>0.006708</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>57</td>
<td>25.24</td>
<td>0.007166</td>
<td>0.007192</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>58</td>
<td>24.42</td>
<td>0.007677</td>
<td>0.007707</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>59</td>
<td>23.60</td>
<td>0.008233</td>
<td>0.008267</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>60</td>
<td>22.80</td>
<td>0.008854</td>
<td>0.008893</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>61</td>
<td>21.99</td>
<td>0.009552</td>
<td>0.009598</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>62</td>
<td>21.20</td>
<td>0.010323</td>
<td>0.010376</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>63</td>
<td>20.42</td>
<td>0.011172</td>
<td>0.011235</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>64</td>
<td>19.64</td>
<td>0.012113</td>
<td>0.012187</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>65</td>
<td>18.88</td>
<td>0.013181</td>
<td>0.013269</td>
<td>117.6939</td>
<td>151.9876</td>
</tr>
<tr>
<td>66</td>
<td>18.12</td>
<td>0.014374</td>
<td>0.014478</td>
<td>115.1577</td>
<td>147.7118</td>
</tr>
<tr>
<td>67</td>
<td>17.38</td>
<td>0.015665</td>
<td>0.015789</td>
<td>112.5519</td>
<td>143.3919</td>
</tr>
<tr>
<td>68</td>
<td>16.65</td>
<td>0.017056</td>
<td>0.017203</td>
<td>109.8756</td>
<td>139.0295</td>
</tr>
<tr>
<td>69</td>
<td>15.93</td>
<td>0.018576</td>
<td>0.018751</td>
<td>107.1273</td>
<td>134.6252</td>
</tr>
<tr>
<td>70</td>
<td>15.22</td>
<td>0.020314</td>
<td>0.020524</td>
<td>104.3072</td>
<td>130.1821</td>
</tr>
<tr>
<td>Age (x_i)</td>
<td>Life Expectancy (years) (e_i)</td>
<td>Death Probability (q_i)</td>
<td>Force-of-Mortality Probability (f_i)</td>
<td>Uniform Monthly Annuity Factors for the First Month of the Year</td>
<td>Inflation-adjusted Monthly Annuity Factors for the First Month of the Year</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------------------------</td>
<td>-------------------------</td>
<td>-------------------------------------</td>
<td>---------------------------------------------------------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>71</td>
<td>14.53</td>
<td>0.02227</td>
<td>0.022529</td>
<td>101.4239</td>
<td>125.7133</td>
</tr>
<tr>
<td>72</td>
<td>13.85</td>
<td>0.024406</td>
<td>0.024708</td>
<td>98.4856</td>
<td>121.2309</td>
</tr>
<tr>
<td>73</td>
<td>13.18</td>
<td>0.026695</td>
<td>0.027058</td>
<td>95.4925</td>
<td>116.7370</td>
</tr>
<tr>
<td>74</td>
<td>12.53</td>
<td>0.029207</td>
<td>0.029642</td>
<td>92.4424</td>
<td>112.2306</td>
</tr>
<tr>
<td>75</td>
<td>11.89</td>
<td>0.032111</td>
<td>0.032638</td>
<td>89.3370</td>
<td>107.7160</td>
</tr>
<tr>
<td>76</td>
<td>11.27</td>
<td>0.035415</td>
<td>0.036057</td>
<td>86.1922</td>
<td>103.2135</td>
</tr>
<tr>
<td>77</td>
<td>10.66</td>
<td>0.038994</td>
<td>0.039774</td>
<td>83.0219</td>
<td>98.7408</td>
</tr>
<tr>
<td>78</td>
<td>10.08</td>
<td>0.042837</td>
<td>0.043781</td>
<td>79.8263</td>
<td>94.2984</td>
</tr>
<tr>
<td>79</td>
<td>9.50</td>
<td>0.047063</td>
<td>0.048206</td>
<td>76.6015</td>
<td>89.8822</td>
</tr>
<tr>
<td>80</td>
<td>8.95</td>
<td>0.051906</td>
<td>0.053301</td>
<td>73.3500</td>
<td>85.4962</td>
</tr>
<tr>
<td>81</td>
<td>8.41</td>
<td>0.057459</td>
<td>0.059175</td>
<td>70.0896</td>
<td>81.1612</td>
</tr>
<tr>
<td>82</td>
<td>7.89</td>
<td>0.063648</td>
<td>0.065763</td>
<td>66.8410</td>
<td>76.9009</td>
</tr>
<tr>
<td>83</td>
<td>7.40</td>
<td>0.070515</td>
<td>0.073124</td>
<td>63.6153</td>
<td>72.7270</td>
</tr>
<tr>
<td>84</td>
<td>6.92</td>
<td>0.078164</td>
<td>0.081388</td>
<td>60.4220</td>
<td>68.6490</td>
</tr>
<tr>
<td>85</td>
<td>6.46</td>
<td>0.086714</td>
<td>0.090706</td>
<td>57.2732</td>
<td>64.6789</td>
</tr>
<tr>
<td>86</td>
<td>6.03</td>
<td>0.096263</td>
<td>0.101217</td>
<td>54.1842</td>
<td>60.8317</td>
</tr>
<tr>
<td>87</td>
<td>5.62</td>
<td>0.106880</td>
<td>0.113035</td>
<td>51.1716</td>
<td>57.1238</td>
</tr>
<tr>
<td>88</td>
<td>5.23</td>
<td>0.118606</td>
<td>0.126251</td>
<td>48.2522</td>
<td>53.5709</td>
</tr>
<tr>
<td>89</td>
<td>4.87</td>
<td>0.131451</td>
<td>0.140931</td>
<td>45.4418</td>
<td>50.1868</td>
</tr>
<tr>
<td>90</td>
<td>4.53</td>
<td>0.145412</td>
<td>0.157136</td>
<td>42.7539</td>
<td>46.9828</td>
</tr>
<tr>
<td>91</td>
<td>4.21</td>
<td>0.160474</td>
<td>0.174918</td>
<td>40.2005</td>
<td>43.9682</td>
</tr>
<tr>
<td>92</td>
<td>3.92</td>
<td>0.176613</td>
<td>0.194329</td>
<td>37.7928</td>
<td>41.1509</td>
</tr>
<tr>
<td>93</td>
<td>3.66</td>
<td>0.193799</td>
<td>0.215422</td>
<td>35.5427</td>
<td>38.5397</td>
</tr>
<tr>
<td>94</td>
<td>3.42</td>
<td>0.211994</td>
<td>0.238250</td>
<td>33.4649</td>
<td>36.1461</td>
</tr>
<tr>
<td>95</td>
<td>3.20</td>
<td>0.230169</td>
<td>0.261584</td>
<td>31.5809</td>
<td>33.9883</td>
</tr>
<tr>
<td>96</td>
<td>3.01</td>
<td>0.248041</td>
<td>0.285079</td>
<td>29.8776</td>
<td>32.0468</td>
</tr>
<tr>
<td>Age ($x_i$)</td>
<td>Life Expectancy (years) ($e_i$)</td>
<td>Death Probability ($q_i$)</td>
<td>Force-of-Mortality Probability ($f_i$)</td>
<td>Uniform Monthly Annuity Factors for the First Month of the Year</td>
<td>Inflation-adjusted Monthly Annuity Factors for the First Month of the Year</td>
</tr>
<tr>
<td>------------</td>
<td>---------------------------------</td>
<td>---------------------------</td>
<td>----------------------------------------</td>
<td>---------------------------------------------------------------</td>
<td>---------------------------------------------------------------------</td>
</tr>
<tr>
<td>97</td>
<td>2.84</td>
<td>0.265318</td>
<td>0.308317</td>
<td>28.3359</td>
<td>30.2968</td>
</tr>
<tr>
<td>98</td>
<td>2.68</td>
<td>0.281695</td>
<td>0.330861</td>
<td>26.9285</td>
<td>28.7046</td>
</tr>
<tr>
<td>99</td>
<td>2.54</td>
<td>0.296871</td>
<td>0.352215</td>
<td>25.6135</td>
<td>27.2223</td>
</tr>
<tr>
<td>100</td>
<td>2.40</td>
<td>0.312977</td>
<td>0.375388</td>
<td>24.3250</td>
<td>25.7781</td>
</tr>
<tr>
<td>101</td>
<td>2.27</td>
<td>0.330077</td>
<td>0.400592</td>
<td>23.0633</td>
<td>24.3718</td>
</tr>
<tr>
<td>102</td>
<td>2.14</td>
<td>0.348236</td>
<td>0.428073</td>
<td>21.8287</td>
<td>23.0032</td>
</tr>
<tr>
<td>103</td>
<td>2.02</td>
<td>0.367528</td>
<td>0.458120</td>
<td>20.6214</td>
<td>21.6719</td>
</tr>
<tr>
<td>104</td>
<td>1.90</td>
<td>0.388029</td>
<td>0.491070</td>
<td>19.4415</td>
<td>20.3777</td>
</tr>
<tr>
<td>105</td>
<td>1.78</td>
<td>0.409816</td>
<td>0.527321</td>
<td>18.2892</td>
<td>19.1201</td>
</tr>
<tr>
<td>106</td>
<td>1.67</td>
<td>0.432975</td>
<td>0.567352</td>
<td>17.1642</td>
<td>17.8985</td>
</tr>
<tr>
<td>107</td>
<td>1.56</td>
<td>0.457593</td>
<td>0.611739</td>
<td>16.0665</td>
<td>16.7122</td>
</tr>
<tr>
<td>108</td>
<td>1.46</td>
<td>0.483763</td>
<td>0.661189</td>
<td>14.9955</td>
<td>15.5605</td>
</tr>
<tr>
<td>109</td>
<td>1.37</td>
<td>0.511581</td>
<td>0.716582</td>
<td>13.9509</td>
<td>14.4422</td>
</tr>
<tr>
<td>110</td>
<td>1.27</td>
<td>0.541150</td>
<td>0.779033</td>
<td>12.9316</td>
<td>13.3561</td>
</tr>
<tr>
<td>111</td>
<td>1.18</td>
<td>0.572575</td>
<td>0.849977</td>
<td>11.9366</td>
<td>12.3004</td>
</tr>
<tr>
<td>112</td>
<td>1.10</td>
<td>0.605968</td>
<td>0.931323</td>
<td>10.9641</td>
<td>11.2733</td>
</tr>
<tr>
<td>113</td>
<td>1.02</td>
<td>0.641446</td>
<td>1.025675</td>
<td>10.0120</td>
<td>10.2718</td>
</tr>
<tr>
<td>114</td>
<td>0.94</td>
<td>0.679129</td>
<td>1.136717</td>
<td>9.0770</td>
<td>9.2926</td>
</tr>
<tr>
<td>115</td>
<td>0.86</td>
<td>0.719145</td>
<td>1.269917</td>
<td>8.1546</td>
<td>8.3307</td>
</tr>
<tr>
<td>116</td>
<td>0.79</td>
<td>0.761624</td>
<td>1.433908</td>
<td>7.2383</td>
<td>7.3792</td>
</tr>
<tr>
<td>117</td>
<td>0.73</td>
<td>0.806699</td>
<td>1.643507</td>
<td>6.3178</td>
<td>6.4282</td>
</tr>
<tr>
<td>118</td>
<td>0.67</td>
<td>0.851378</td>
<td>1.906349</td>
<td>5.3799</td>
<td>5.4680</td>
</tr>
<tr>
<td>119</td>
<td>0.61</td>
<td>0.893947</td>
<td>2.243816</td>
<td>4.0607</td>
<td>4.1685</td>
</tr>
</tbody>
</table>