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The Social Value of Mortality Risk Reduction: VSL vs. the Social Welfare Function Approach

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Abstract

We examine how different welfarist frameworks evaluate the social value of mortality risk-reduction. These frameworks include classical, distributively unweighted cost-benefit analysis—i.e., the "value per statistical life" (VSL) approach—and three benchmark social welfare functions (SWF): a utilitarian SWF, an ex ante prioritarian SWF, and an ex post prioritarian SWF. We examine the conditions on individual utility and on the SWF under which these frameworks display the following five properties: i) wealth sensitivity, ii) sensitivity to baseline risk, iii) equal value of risk reduction, iv) preference for risk equity, and v) catastrophe aversion. We show that the particular manner in which VSL ranks risk-reduction measures is not necessarily shared by other welfarist frameworks, and we identify when the use of an ex ante or an ex post approach has different implications for risk policymaking.

Keywords: Value of statistical life, social welfare functions, cost-benefit analysis, equity, fairness, welfarism, risk policy.

JEL: D81, D61, D63, Q51.

Introduction

Two, quite different intellectual traditions exist concerning cost-benefit analysis (CBA). One view, dominant in the United States, sees CBA as a way to identify projects that pass a Kaldor-Hicks compensation test and advocates summing unweighted compensating or equivalent variations. Another approach, influential in the U.K. and Europe, sees the "social welfare function" (SWF) as the fundamental basis for policymaking.[1] CBA can generally mimic the effect of a SWF if compensating or equivalent variations are multiplied by distributive weights that reflect the declining marginal utility of wealth and also, perhaps, social inequality aversion.

Scholarship regarding the "value per statistical life" (VSL) has, almost invariably, taken the Kaldor-Hicks approach. VSL is the marginal rate of substitution between fatality risk in a specified time period, and wealth. In other words, it is the (distributively unweighted) change in an individual's wealth required to compensate him for a small change in his risk of dying during the period, divided by the risk change.

The social value of mortality risk-reduction depends on our moral assumptions about risk and equity. American law (Executive Orders 12866 and 13563) instructs regulatory agencies to be sensitive to equity. Yet VSL has properties that can yield what are often viewed as inequitable evaluations of policy change. In particular, VSL does not value reductions in mortality risk equally. In some dimensions it favors those who are better off (individuals with higher wealth). In other dimensions, it favors the less well-off (individuals at higher risk of dying). But how does VSL generally compare with other frameworks?

This article formally examines the social value of mortality-risk reduction through the lens of a SWF. It asks: to what extent are the properties of VSL characteristic of various welfarist frameworks? If one views some of the implications of using VSL to value risk policies as inequitable, is there an SWF that exhibits a more attractive set of implications? In short, what happens if we shift from distributively unweighted CBA to some SWF as the societal tool for evaluating risk reductions?

Part I reviews the SWF approach. As recent scholarship has demonstrated, how to apply an SWF under uncertainty is an interesting and difficult problem. There are a wide range of possibilities, each with axiomatic advantages and disadvantages.

Part II discusses how three "benchmark" SWFs evaluate risk reduction—comparing them with each other, and with conventional CBA using VSL. The three benchmarks are: a utilitarian SWF, an "ex post" prioritarian (additively separable and strictly concave) SWF; and an "ex ante" prioritarian SWF. In order to make the comparison tractable, we use the standard, one-period

[1]To be sure, the concept of the SWF is hardly absent from scholarly discourse in the U.S. For example, it plays a central role in scholarship regarding optimal taxation. See, e.g., Kaplow (2008). However, it has been largely absent from U.S. scholarship and governmental practice regarding CBA.

model that is familiar from the VSL literature. We focus on five properties that have attracted interest in the economic and risk policy literature:

(1) *Wealth sensitivity*: Does the social value of risk reduction—as calculated by CBA or by the benchmark SWFs—increase with individual wealth? As is well known, VSL increases with wealth but cross-sectional differences in VSL attributable to wealth are virtually always suppressed in policy evaluation. Public /political resistance to differentiating VSL by wealth is so strong that use of a different (higher) VSL was rejected in a context where both the costs and benefits of regulation would fall on an identified higher-income group (airline passengers; Viscusi 2009). In contrast, increases in VSL attributable to future income growth are often incorporated in analyses (Robinson 2007). As we shall see, the social value of risk reduction increases with wealth for CBA and for the utilitarian SWF. By contrast, the ex ante and ex post prioritarian SWFs need not be positively sensitive to individual wealth in valuing risk reduction.

(2) *Sensitivity to baseline risk*. Does the social value of risk reduction depend on the individual's baseline risk of dying? This property, the "dead-anyway effect" (Pratt and Zeckhauser 1996) is not only of intrinsic interest, but is closely connected to the problem of statistical versus identified lives (Hammitt and Treich 2007) and to the "rule of rescue", a moral imperative to give life-saving priority to people at high risk (Jonsen 1986). In contrast, we note that it has been recommended in some policy circles to not adjust the value of lifesaving programs for the health status of the affected population (European Commission 2001). The social value of risk reduction increases with baseline risk for CBA and the ex ante prioritarian SWF, but is independent of baseline risk for the utilitarian and ex post prioritarian SWFs.

(3) *Equal value of risk reduction*. This property requires insensitivity to both wealth and baseline risk, as well as to other factors such as age and health. The nearly ubiquitous use of a common VSL by a federal agency, the interest in harmonizing VSLs among agencies (e.g., U.S. Office of Management and Budget 2003, HM Treasury 2011) or countries (e.g., European Commission 2001, Fankhauser et al. 1997), and the adverse reaction to using a different (smaller) VSL for older people in EPA air regulations (Viscusi 2009), are consistent with widespread interest in equal value of risk reduction. Equal value of risk reduction is obviously not satisfied by CBA, nor is it satisfied by any of the benchmark SWFs (except by the ex post prioritarian SWF under restrictive parameter assumptions).

(4) *Risk equity preference*. If a policy *equalizes* individuals' risks of dying, does that count as a social improvement? A preference for risk equity was discussed by Keeney (1980) and is reflected in concerns for environmental justice (Lazarus 1993). Risk equity is closely related to sensitivity to baseline risk. Equalizing risk improves social welfare under the ex ante prioritarian SWF and under CBA using equivalent variations (though not necessarily when using compensating variations), but not under the utilitarian and ex post prioritarian SWFs.

(5) *Catastrophe aversion*. If a policy does not change the expected number of deaths, but reduces the chance of multiple individuals dying, does that count as a social improvement? It is widely noted that incidents in which many people die (e.g., an airliner crash or a nuclear disaster) are regarded as worse than an equal number of fatalities in unrelated events (e.g., traffic crashes or heart attacks). Keeney (1980) identifies catastrophe aversion as an attractive property, and catastrophic potential appears to be a major determinant of risk perceptions (Slovic 2000). However, neither CBA nor any of the benchmark SWFs are consistent with catastrophe aversion.

Results for these five properties are summarized in Table 1, below.

We examine the properties of equal value of risk reduction and catastrophe aversion in more detail in Parts III and IV, respectively. Many seem to find one or both of these properties desirable in a risk-evaluation tool; and yet the latter property is not satisfied by either VSL or any of the benchmarks; the former, only by one of the benchmarks, and under restrictive assumptions. We therefore ask: if we introduce a different kind of SWF, or relax standard assumptions regarding the form of individual utility, are these properties satisfied? In Part III, we criticize the weighted utilitarian SWF suggested by Baker et al. (2008); but we concur in their suggestion that equal value of risk reduction might plausibly be achieved by combining the utilitarian SWF with a utility function that equalizes marginal utility of individual wealth when an individual is dead to its marginal utility when he is alive. In Part IV, we find that ex post "transformed" utilitarian and prioritarian SWFs—unlike CBA or any of the benchmark SWFs—satisfy catastrophe aversion with a concave "transformation" function.

Our analysis puts CBA using VSL in a new light. The particular manner in which VSL ranks risk-reduction measures is not the *inevitable* result of a welfarist approach to policymaking. VSL's salient features can, if seen as undesirable, be mitigated by shifting to some social welfare function. However, we have not identified an SWF satisfying all of the properties that might plausibly be viewed as desirable.

Short proofs are provided in the text or footnotes, with longer proofs generally relegated to an Appendix.

I. SWFs Under Uncertainty

The SWF approach assumes some interpersonally comparable function $\mathbf{u}(.)$. If x is an outcome, then $\mathbf{u}(x) = (u_1(x), \ldots., u_N(x))$, with N individuals in the population.[2] (Throughout this article, we assume that N is the same in all outcomes.) An SWF is a rule R for ranking outcomes as a function of their associated utility vectors. It says: $x \succcurlyeq y$ iff $\mathbf{u}(x)\ R\ \mathbf{u}(y)$ (where "iff" means

[2] The function $\mathbf{u}(.)$ is a functional. The ith argument of $\mathbf{u}(x)$, denoted $u_i(x)$, represents the well-being level of individual i in outcome x. Function $\mathbf{u}(.)$ is "interpersonally comparable" in the sense that these numbers represent well-being levels and differences as compared between different persons. For example, $u_i(x) > u_j(y)$ iff individual i in outcome x is better off than individual j in outcome y. On interpersonal comparability, see generally Adler (2012, Chapters 2 and 3).

“if and only if”). The literature discusses standard forms for R. One is a utilitarian SWF: $x \succcurlyeq y$ iff $\sum_{i=1}^{N} u_i(x) \geq \sum_{i=1}^{N} u_i(y)$. Another is a “prioritarian” (additively separable, concave SWF): $x \succcurlyeq y$ iff $\sum_{i=1}^{N} g(u_i(x)) \geq \sum_{i=1}^{N} g(u_i(y))$, with $g(.)$ a strictly increasing and concave real-valued function. A third is the “leximin” SWF, which ranks utility vectors according to their smallest entries, if these are equal their second-smallest, etc. Finally, the “rank-weighted” SWF uses fixed weights $\alpha_1 > \alpha_2 \ldots > \alpha_N$, with α_1 the weight for the smallest utility in a vector, α_2 the second smallest, etc., and ranks vectors by summing weighted utilities.[3] (On the different functional forms of an SWF, see generally Adler 2012, Bossert and Weymark 2004, Blackorby et al. 2005.)

As recent scholarship has shown, a wide range of possibilities exist for applying an SWF under uncertainty, with different axiomatic characteristics (Fleurbaey 2010, Adler 2012, Chapter 7). In representing policy choice under uncertainty, we will use a standard Savage-style model where there is a set of states and a fixed probability assigned to each state s, π_s. An action (e.g., governmental policy) maps each state onto an outcome. Let $x^{a,s}$ be the outcome of action a in state s.

Consider, first, the possibilities for a utilitarian SWF. “Ex post” *untransformed* utilitarianism assigns each action a number equaling the expected value of the sum of individual utilities. In other words, $a \succcurlyeq b$ iff $W(a) \geq W(b)$, with $W(a) = \sum_s \pi_s \sum_i u_i(x^{a,s})$. “Ex post” untransformed utilitarianism yields the same ranking of actions as “ex ante” utilitarianism, ranking actions according to the sum of individual expected utilities. Let $U_i(a) = \sum_s \pi_s u_i(x^{a,s})$. Then “ex ante” utilitarianism says: $a \succcurlyeq b$ iff $W(a) \geq W(b)$, with $W(a) = \sum_i U_i(a)$.

Ex post utilitarianism can also take a “transformed” form. Let $h(.)$ be a strictly increasing (but not necessarily linear) function. Then ex post *transformed* utilitarianism sets $W(a) = \sum_s \pi_s h(\sum_i u_i(x^{a,s}))$. Note that, if $h(.)$ is non-linear, ex post transformed utilitarianism need not rank actions the same way as ex ante utilitarianism.

Consider, next, the possibilities for a prioritarian SWF. “Ex post” *untransformed* prioritarianism assigns each action a number equaling the expected value of the sum of a strictly increasing and concave function of individual utility. In other words, $a \succcurlyeq b$ iff $W(a) \geq W(b)$,

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[3] Let $\tilde{u}_1(x) \leq \tilde{u}_2(x) \leq \ldots \leq \tilde{u}_N(x)$ denote a rank-ordered permutation of the vector $\mathbf{u}(x)$. Then the rank-ordered SWF ranks outcomes as follows, using some fixed set of strictly decreasing weights $\alpha_1, \ldots, \alpha_N$: $x \succcurlyeq y$ iff $\sum_i \alpha_i \tilde{u}_i(x) \geq \sum_i \alpha_i \tilde{u}_i(y)$, with x and y two outcomes.

with $W(a) = \sum_s \pi_s \sum_i g(u_i(x^{a,s}))$. While ex post untransformed utilitarianism is mathematically equivalent to ex ante utilitarianism, ex post untransformed prioritarianism is not equivalent to "ex ante" prioritarianism, where $W(a) = \sum_i g(U_i(a))$. Finally, ex post *transformed* prioritarianism should be mentioned: $W(a) = \sum_s \pi_s h(\sum_i g(u_i(x^{a,s}))$, with $h(.)$ strictly increasing.

Fleurbaey (2010) focuses on the properties of a particular kind of ex post transformation: the "equally distributed equivalent" (EDE). Let $w(.)$ be a function from utility vectors to real numbers corresponding to the utilitarian, prioritarian, or rank-weighted SWF, as the case may be.[4] Let u^* be such that $w(u^*, u^*, \ldots, u^*) = w(\mathbf{u}(x))$ for a given outcome x. Define the real-valued function $h^{EDE}(.)$ as follows: $h^{EDE}(w(\mathbf{u}(x)) = u^*$. In the case of the utilitarian SWF, $h^{EDE}(.)$ is just average utility: $h^{EDE}(\sum_i u_i(x)) = (1/N)\sum_i u_i(x)$, i.e., $h^{EDE}(w) = w/N$. In this case, $h^{EDE}(.)$ is a linear function. By contrast, in the case of the prioritarian SWF, $h^{EDE}(.)$ is strictly *convex*. Note that $h^{EDE}\left[\sum_i g(u_i(x))\right] = g^{-1}\left[(1/N)\sum_i g(u_i(x))\right]$, i.e., $h^{EDE}(w) = g^{-1}(w/N)$, leading to $W(a) = \sum_s \pi_s g^{-1}(\sum_i g(u_i(x^{a,s}))$

To keep the analysis tractable, we will not consider the rank-weighted SWF or the leximin SWF. Instead, our focus will be on different possible methodologies for applying a utilitarian or prioritarian SWF to value risk-reduction measures.[5]

II. **VSL versus SWF: A Simple Model**

For the remainder of the paper, unless otherwise noted, we use "CBA" to mean cost-benefit analysis without distributive weights.[6] CBA ranks policies by summing equivalent or compensating variations. As is well known, CBA does not provide a social ranking—it can violate completeness and transitivity (Blackorby and Donaldson 1986). However, we can use CBA to define a social ranking of alternatives using equivalent or compensating variation from a

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[4] In the case of the utilitarian SWF, $w(\mathbf{u}(x)) = \sum_i u_i(x)$; in the case of the prioritarian SWF, $w(\mathbf{u}(x)) = \sum_i g(u_i(x))$; and the rank-weighted SWF, $w(\mathbf{u}(x)) = \sum_i \alpha_i \tilde{u}_i(x)$.

[5] Some authors, e.g., Ben Porath et al. (1997), Ulph (1982), have characterized a "hybrid" approach. Let $W(a)$ be the value assigned to an action by ex post (transformed or untransformed) utilitarianism, prioritarianism, or the rank-weighted approach, and $W^*(a)$ the value assigned by, respectively, ex ante utilitarianism, prioritarianism, or the rank weighted approach. Then if λ is between 0 and 1, the hybrid approach assigns each action a value equaling $\lambda W(a) + (1 - \lambda) W^*(a)$. This approach, too, is beyond the scope of the current article.

[6] Most SWFs can be mimicked by CBA with appropriate distributive weights (Adler 2012, pp. 109-10, Drèze and Stern 1987). Since the theme of this article is the divergence between SWFs and traditional, unweighted, CBA, we do not address the specification of weights here.

fixed baseline. Consider some baseline action *O*, the "status quo" action. Let *a*, *b*, … be other possible actions (governmental policies). For a given such action *a*, let individual *i*'s equivalent variation EV_i^a be the change to individual *i*'s wealth in every state of the world, in *O*, sufficient to make *i* ex ante indifferent as between *O* and *a*.[7] Then we will say that CBA ranks actions by saying: $a \succcurlyeq b$ iff $W^{CBA}(a) \geq W^{CBA}(b)$, where $W^{CBA}(a) = \sum_i EV_i^a$.

In order to compare CBA to various SWFs, we adopt the following simple, one-period model—one that is frequently used in the discussion of VSL. Each policy (*a*, *b*, …) is such that each individual has the same wealth (c_i^a, c_i^b, etc.) in all states as a result of that policy (although *not* necessarily the same across policies or individuals.) For a given policy, the state determines which individuals will be alive or dead. We introduce $l_i^{a,s}$, which has the value 1 if individual *i* is alive and 0 if dead. Utility functions $u(.)$ and $v(.)$ are the (common and interpersonally comparable) utility functions of wealth if individuals are alive and dead, respectively (i.e., $v(.)$ is the bequest function).

We assume, as is standard in the VSL literature, that $u(c) > v(c)$, $u'(c) > v'(c) \geq 0$ and $u''(c) \leq 0$, $v''(c) \leq 0$. We refer to this package of assumptions as the "standard" utility model (although it should be recognized that the assumptions are not *entailed* by expected utility theory; we relax some of them in Part III.B).

Let p_i^a be individual *i*'s probability of being alive with policy *a*, that is, $p_i^a = \sum_s \pi_s l_i^{a,s}$. Then $U_i(a)$, individual *i*'s expected utility with action *a*, is simply $p_i^a\, u(c_i^a) + (1 - p_i^a)\, v(c_i^a)$.

Some of our results in this section depend upon a zero bequest function, i.e., $v(c) = 0$ for all *c*. Note that this is consistent with the standard utility model.[8]

We focus in this Part on three "benchmark" SWFs: ex post untransformed utilitarianism (which is equivalent, recall, to ex ante utilitarianism); ex post untransformed prioritarianism, and

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[7] Alternatively, one can construct a ranking using the sum of compensating variations from a fixed baseline where individual *i*'s compensating variation CV_i^a is the change to individual *i*'s wealth in every state of the world, in *a*, sufficient to make *i* ex ante indifferent as between *O* and *a*, CV_i^b is the analogous change to individual *i*'s wealth in every state of the world, in *b*, and so forth. The social ranking based on compensating variation can violate the Pareto principle, while the social ranking based on equivalent variation cannot. The reason is that the individual's marginal utility of wealth can depend on the state of the world (e.g., if he lives or dies). An individual may prefer *a* to *b*, but if his marginal utility of wealth in *a* exceeds his marginal utility of wealth in *b*, CV_i^a can be smaller than CV_i^b. If no one else in the population is affected by shifting from the status quo to *a* or to *b*, then *a* is Pareto superior to *b* yet CBA using compensating variation will rank *b* superior to *a*. This situation cannot arise using the social ranking based on equivalent variation from a fixed baseline, which always adds wealth to the states associated with the same action (the status quo action *0*).

[8] In distinguishing between the case where $v(c) = 0$ and $v(c) \neq 0$, we are assuming that the common, interpersonally comparable utility function $u^*(c, l)$ that gives rise to $u(.)$ and $v(.)$—*l* an indicator variable indicating whether the individual is alive or dead— is unique up to a positive ratio transformation, not merely a positive affine transformation. Prioritarian SWFs, indeed, make stronger assumptions on the measurability of utility than utilitarianism.

ex ante prioritarianism. Because ex post *transformed* utilitarianism and prioritarianism are not introduced until Part III, we can without confusion omit the adjective "untransformed" and refer to ex post untransformed utilitarianism as, simply, "utilitarianism"; and to ex post untransformed prioritarianism as, simply, "ex post prioritarianism."

Utilitarianism, ex post prioritarianism, and ex ante prioritarianism are not only very widely used, but—in the case of the simple one-period model now under discussion—can be expressed in a simple way. As already discussed, each of these approaches (like CBA) ranks policies via a rule of the form: $a \succcurlyeq b$ iff $W(a) \geq W(b)$. Let W^U, W^{EPP}, and W^{EAP} denote the W-functions associated, respectively, with these three approaches. Then the following can be straightforwardly established:

$$W^U(a) = \sum_i U_i(a)$$

$$W^{EPP}(a) = \sum_i \left[p_i^a g(u(c_i^a)) + (1 - p_i^a) g(v(c_i^a)) \right]$$

$$W^{EAP}(a) = \sum_i g(U_i(a))$$

Note that, in each case—as with W^{CBA}—the ranking of policies depends only on each individual's wealth c_i and mortality probability p_i, and not on the states in which each person is alive or dead, $l_i^{a,s}$.[9]

In the case where policies represent a small variation in individual risk and/or wealth around the status quo policy O, we can use the total differential to approximate a change in W^{CBA}, W^U, W^{EPP}, and W^{EAP}. As a shorthand, and without risk of confusion, we will use the term p_i to mean individual i's survival probability in the status quo (strictly, p_i^O); c_i to mean individual i's wealth in the status quo (strictly, c_i^O); U_i her expected utility in the status quo; and a function incorporating these terms (such as $\partial U_i / \partial p_i$) to mean the function evaluated at the status quo values (here, $\partial U_i / \partial p_i$ evaluated at the values U_i^O and p_i^O).

Consider, now, some policy a that changes each individual i's survival probability by Δp_i^a and her wealth by Δc_i^a. Then it can be seen that:[10]

$$\Delta W^{CBA}(a) \approx \sum_i [\Delta c_i^a + VSL_i \Delta p_i^a],$$

[9] This is not true for ex post transformed utilitarianism or prioritarianism with a non-linear transformation function, for which the ranking may depend on how many people are alive in each state (see Part IV).

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[10] Let $\Delta W^{CBA}(a) = W^{CBA}(a) - W^{CBA}(O)$, and similarly for the other approaches.

where VSL_i is individual i's marginal rate of substitution between survival probability and wealth in O, i.e., $\frac{\partial U_i / \partial p_i}{\partial U_i / \partial c_i}$, which equals $\frac{u(c_i) - v(c_i)}{p_i u'(c_i) + (1 - p_i) v'(c_i)}$. Similarly,

$$\Delta W^U(a) \approx \sum_i [\Delta c_i^a [p_i u'(c_i) + (1 - p_i) v'(c_i)] + \Delta p_i^a [u(c_i) - v(c_i)]]$$

$$\Delta W^{EPP}(a) \approx \sum_i [\Delta c_i^a [p_i g'(u(c_i)) u'(c_i) + (1 - p_i) g'(v(c_i)) v'(c_i)] + \Delta p_i^a [g(u(c_i)) - g(v(c_i))]]$$

$$\Delta W^{EAP}(a) \approx \sum_i [\Delta c_i^a [g'(U_i)(p_i u'(c_i) + (1 - p_i) v'(c_i)] + \Delta p_i^a [g'(U_i)(u(c_i) - v(c_i))]]$$

It is useful to think of W^{CBA}, W^U, W^{EPP}, and W^{EAP} as different methodologies for assigning a "social value" to policies. Note that, in each case, the total-differential approximation allows us to distinguish (1) the change in "social value" associated with the change in individual i's wealth (Δc_i^a) from (2) the change in "social value" associated with the change in her survival probability (Δp_i^a). The latter change is just $(\partial W / \partial p_i)\Delta p_i^a$. For short, let us say that the social value of risk reduction, for a given individual i, according to a given W, is just $\partial W / \partial p_i$.[11] (To be clear, this social value may well depend upon i's wealth in the status quo c_i, her survival probability p_i, or both.) Note that, in the case of CBA, the social value of risk reduction is simply VSL_i.

We now turn to the central question of this Part. How do these different approaches *compare* in assigning social value to risk reduction? In particular, in the status quo, individual i has wealth c_i and survival probability p_i, while individual j has a different amount of wealth c_j and/or a different survival probability p_j. How does the social value of risk reduction for the first individual, $\partial W / \partial p_i$, compare with the social value of risk reduction for the second, $\partial W / \partial p_j$—with "social value" calculated using W^{CBA} or, alternatively, W^U, W^{EPP}, and W^{EAP}?

Note that the social value of risk reduction is positive for all of the W functions considered here, regardless of individual wealth and baseline risk. Thus $\frac{\partial W}{\partial p_i} > \frac{\partial W}{\partial p_j}$ iff $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$ > 1. In what follows, we often focus on the ratio $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$.[12]

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[11] Remember that each policy is associated with values of p_i and c_i for each individual; that W in the case of W^U, W^{EPP}, and W^{EAP} is a mapping from each policy onto a real number as a function of the p_i and c_i values for all individuals; and that $\partial W / \partial p_i$ is shorthand for the function $\partial W / \partial p_i$ evaluated at individual i's status quo survival probability, p_i^O, and wealth c_i^O.

[12] In this article, we are interested in the *ordinal* properties of the different W functions (W^{CBA}, W^U, W^{EPP}, and W^{EAP}), i.e., the ordinal ranking of policies that they generate. Our interest in the ratio just described is *consistent* with the

A. *Wealth Sensitivity*

Consider the case where individual i has more status quo wealth than individual j ($c_i > c_j$) and both have the same survival probability $p_i = p_j$. This set-up allows us to isolate the effect of individual wealth on the social value of individual risk reduction. We define a social ranking as (positively) wealth sensitive if it always assigns higher value to reducing risk to the wealthier of two individuals having the same mortality risk.

Definition 1: Let $c_i > c_j$ and $p_i = p_j$. A social ranking is (positively) *wealth sensitive* iff $\frac{\partial W}{\partial p_i} > \frac{\partial W}{\partial p_j}$**.**

The following equations summarize the social value of risk reduction for CBA as compared with utilitarianism, ex post prioritarianism, and ex ante prioritarianism.

$$\frac{\partial W^{CBA}}{\partial p_i} = VSL_i = \frac{u(c_i) - v(c_i)}{p_i u'(c_i) + (1 - p_i)v'(c_i)}$$

$$\frac{\partial W^{U}}{\partial p_i} = u(c_i) - v(c_i)$$

$$\frac{\partial W^{EPP}}{\partial p_i} = g(u(c_i)) - g(v(c_i))$$

$$\frac{\partial W^{EAP}}{\partial p_i} = g'(U_i)(u(c_i) - v(c_i))$$

CBA is (positively) sensitive to individual wealth. As is well known, CBA assigns the wealthier individual a greater social value of individual risk reduction: $VSL_i/VSL_j > 1$. The same is true of utilitarianism: $\frac{u(c_i) - v(c_i)}{u(c_j) - v(c_j)} > 1$, on the assumption that $u'(.) > v'(.)$.

However, ex post and ex ante prioritarianism do *not* necessarily assign the wealthier individual a greater social value of risk reduction. In the case of ex post prioritarianism, the relevant ratio is $\frac{g(u(c_i)) - g(v(c_i))}{g(u(c_j)) - g(v(c_j))}$. In the case of ex ante prioritarianism, it is

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fact that W merely has ordinal significance. Let $f(.)$ be any differentiable, strictly increasing function. Then $\frac{\partial f(W)}{\partial p_i} > \frac{\partial f(W)}{\partial p_j}$ iff $f'(W)\frac{\partial W}{\partial p_i} > f'(W)\frac{\partial W}{\partial p_j}$ iff $\frac{\partial W / \partial p_i}{\partial W / \partial p_j} > 1$, since $f'(.) > 0$ as are $\partial W / \partial p_i$ and $\partial W / \partial p_j$.

$\dfrac{g'(U_i)(u(c_i)-v(c_i))}{g'(U_j)(u(c_j)-v(c_j))}$. With $c_i > c_j$, these ratios can be greater than, less than, or equal to one, depending on the functional forms of $g(.)$, $u(.)$, and $v(.)$.[13] We therefore arrive at our first result.

PROPOSITION I: CBA and utilitarianism are (positively) wealth sensitive: the social value of individual risk reduction increases with individual wealth. In the case of ex post prioritarianism and ex ante prioritarianism, the social value of individual risk reduction can increase with individual wealth, decrease with individual wealth, or remain neutral—depending on the functional forms of *g*(.), *u*(.), and *v*(.).

This is an important result. CBA's positive wealth sensitivity in valuing risk reduction does not emerge as a general feature of welfarism (even if we confine our attention to the three benchmark SWFs, let alone other SWFs). Although VSL_i increases with individual wealth, that is not necessarily true of $\dfrac{\partial W^{EPP}}{\partial p_i}$ or $\dfrac{\partial W^{EAP}}{\partial p_i}$.

Can we achieve clearer results regarding the wealth sensitivity of ex post and ex ante prioritarianism by restricting the bequest function to be zero ($v(.) = 0$)? With a zero bequest function, ex post prioritarianism is only well-defined if $g(0)$ is well-defined.[14] Continuing to focus on the case where $c_i > c_j$ and $p_i = p_j$, the ratio $\dfrac{\partial W^{EPP}/\partial p_i}{\partial W^{EPP}/\partial p_j}$ becomes $\dfrac{g(u(c_i))-g(0)}{g(u(c_j))-g(0)}$, which *is* greater than unity, since $g(.)$ and $u(.)$ are strictly increasing.

However, even with a zero bequest function, ex ante prioritarianism may be insensitive to wealth in valuing risk reduction, or may give less weight to risk reduction as individuals become wealthier. Note that the ratio $\dfrac{\partial W^{EAP}/\partial p_i}{\partial W^{EAP}/\partial p_j}$ becomes $\dfrac{g'(U_i)u(c_i)}{g'(U_j)u(c_j)}$. Setting $g(.) = \log$ makes this ratio unity. Moreover, if the $g(.)$ function is more concave than the logarithm, ex ante prioritarianism is *negatively* wealth-sensitive—assigning a *lower* social value to risk reduction for wealthier individuals.[15]

[13] Consider, first, ex post prioritarianism. The ratio is greater than one in the case of a zero bequest function, the case considered immediately below. Alternatively, let $v(.) = ku(.)$ with $0< k< 1$. With $g(x) = \log x$, the ratio is unity while with $g(x) = -1/x$ for instance, the ratio is less than one. Next consider ex ante prioritarianism. As discussed immediately below, the ratio can be greater than, less than, or equal to zero even if the bequest function is constrained to be zero, and *a fortiori* without such constraint.
[14] This rules out strictly increasing, strictly concave $g(.)$ functions with $g(0) = -\infty$, such as the log function, or $-(1/x)^\gamma$ with $\gamma > 0$.
[15] Let $F(c) = g'(pu(c))u(c)$. Then $\dfrac{g'(U_i)u(c_i)}{g'(U_j)u(c_j)} > 1$ (resp. < 1) for any $c_i > c_j$ with a zero bequest function reduces to $F'(.) > 0$ (resp. < 0) for all c. But note that $F'(c) > 0$ for all c just in case $-xg''(x)/g'(x) > 1$ for all x, i.e., $g(.)$ has a

A different aspect of the problem of wealth sensitivity concerns whether CBA has a greater *degree* of wealth sensitivity than competing approaches. We show (in the Appendix) that CBA has a greater degree of wealth sensitivity than both utilitarianism and ex ante prioritarianism. In addition, it has a higher degree of wealth sensitivity than ex post prioritarianism with a zero bequest function.

B. *Sensitivity to Baseline Risk*

Next, we isolate the effect of individual survival probability on the social value of risk reduction—by considering a case where individual *i* has survival probability p_i in the status quo, individual *j* has survival probability p_j, with $p_i > p_j$, and the two individuals have the same wealth. We define (positive) sensitivity to baseline risk as follows:

Definition 2: Let $p_i > p_j$ and $c_i = c_j$. A social ranking is (positively) *sensitive to baseline risk* iff $\frac{\partial W}{\partial p_i} < \frac{\partial W}{\partial p_j}$.

As is well-known, $VSL_i/VSL_j < 1$; hence CBA accords a higher social value to individual risk reduction for individuals at lower survival probability . This is the so-called "dead anyway" effect (Pratt and Zeckhauser 1996). Ex ante prioritarianism also displays the dead-anyway effect: $\frac{\partial W^{EAP} / \partial p_i}{\partial W^{EAP} / \partial p_j}$ simplifies to $g'(U_i)/g'(U_j)$ in the case at hand, which is less than unity because $U_i > U_j$ and $g(.)$ is strictly concave, i.e., g' strictly decreasing. By contrast, for ex post prioritarianism and utilitarianism, the social value of risk reduction is insensitive to baseline risk. Note that $\partial W^U / \partial p_i$ and $\partial W^{EPP} / \partial p_i$ are, each, solely a function of *i*'s wealth; and thus $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$ is, in each case, unity where *i* and *j* have the same wealth, regardless of their survival probabilities.

PROPOSITION II: CBA and ex ante prioritarianism are (positively) sensitive to baseline risk. By contrast, utilitarianism and ex post prioritarianism are insensitive to baseline risk.

Scholarship on risk reduction often discusses whether a preference for aiding "identified" rather than "statistical" victims is justified. We might say that an individual is an "identified" victim if her probability of surviving the current period, absent governmental intervention, is zero or (more generally) sufficiently low. An immediate implication of the last paragraph is that CBA and ex ante prioritarianism, but not utilitarianism or ex post prioritarianism, display a preference for aiding identified victims. Concerns about environmental justice and cumulative risk are also consistent with a social value of risk reduction that is increasing with the

degree of concavity globally less than unity; that $F'(c) < 0$ for all c just in case $-xg''(x)/g'(x) < 1$ for all x; and that $-xg''(x)/g'(x) = 1$ if $g(x) = \log x$.

individual's baseline risk, at least to the extent that the baseline risk is determined by environmental exposures.

C. Equal Value of Risk Reduction

As discussed above, there is often reluctance to assign different social values to reducing risk to people with different characteristics. In the simple model we consider, individuals are identical except for any differences in their wealth c or survival probability p. Hence equal value of risk reduction requires insensitivity to both wealth and baseline risk.

Definition 3: A social ranking satisfies *equal value of risk reduction* iff $\frac{\partial W}{\partial p_i} = \frac{\partial W}{\partial p_j}\ \forall\ p_i, p_j, c_i, c_j$.

$W(.)$ displays equal value of risk reduction iff $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$ is unity regardless of the survival probabilities *and* wealth of i and j. As we have seen, CBA and the utilitarian SWF are wealth sensitive, while the ex ante prioritarian SWF (as well as CBA) is sensitive to baseline risk. It thus follows immediately that CBA, the utilitarian SWF, and the ex ante prioritarian SWF do *not* exhibit equal value of risk reduction. By contrast, ex post prioritarianism *can* satisfy equal value of risk reduction—but only under restrictive assumptions regarding $g(.)$ and individual utility.[16]

PROPOSITION III: Neither CBA, nor utilitarianism, nor ex ante prioritarianism satisfy equal value of risk reduction. Under all these approaches, the social value of individual risk reduction depends upon the individual's wealth, survival probability, or both. Ex post prioritarianism satisfies equal value of risk reduction only under restrictive assumptions regarding *g*(.) and individual utility.

D. Risk Equity Preference

A policymaking methodology has a preference for risk equity if it prefers to equalize survival probabilities. Imagine that, in the baseline, individual j has a lower survival probability than individual i: $p_j < p_i$. A policy increases individual j's survival probability to $p_j + \Delta p$, and decreases individual i's survival probability to $p_i - \Delta p$, leaving j still at a survival probability no higher than i. (In other words, the policy secures a Pigou-Dalton transfer in survival probability.) The policy does not change other individuals' survival probabilities, or anyone's wealth. Then we say: (1) a policymaking methodology has a *weak* preference for risk equity if it prefers the policy to baseline as long as i and j have the same wealth; and (2) a policymaking methodology has a *strong* preference for risk equity if it prefers the policy to baseline regardless of the wealth of the two individuals.

[16] Let $F(c) = g(u(c)) - g(v(c))$. Then it is easy to see that ex post prioritarianism satisfies equal value of risk reduction iff $F'(c) = 0$. A sufficient (but not necessary) condition for this to be true is $v(.) = ku(.)$, with $0<k<1$ and $g(x) = \log x$.Note that ex post prioritarianism with a zero bequest function exhibits wealth sensitivity (as discussed earlier) and therefore fails to satisfy equal value of risk reduction.

Definition 4: Let $p_i' = p_i - \Delta p > p_j' = p_j + \Delta p$, with $\Delta p > 0$. Consider a policy a leading to (p_i', p_j') and a policy b leading to (p_i, p_j) while leaving unaffected the survival probabilities and wealth levels of all individuals in society excluding *i* and *j*. A social ranking satisfies a *weak preference for risk equity* iff $a \succ b$ for $c_i = c_j$ and a *strong preference for risk equity* iff $a \succ b$ holds $\forall$ c_i, c_j.

It is immediate from the definitions that weak preference for risk equity is closely related to (positive) sensitivity to baseline risk. Indeed, the preference relationship in the definition of weak risk equity preference is satisfied for infinitesimal Δp if and only if the social ranking is positively sensitive to baseline risk. Hence sensitivity to baseline risk is a necessary condition for risk equity. As we have seen, utilitarianism and ex post prioritarianism are insensitive to baseline risk and hence they do not exhibit a preference for risk equity.

Interestingly, CBA exhibits risk equity preference in the weak sense when the social ranking is defined using equivalent variation, but may not satisfy it using compensating variation. A proof is provided in the Appendix. The difference arises because the effect of wealth on VSL augments the difference between individuals in the case of equivalent variation, but offsets and can reverse the difference in the case of compensating variation.

Ex ante prioritarianism satisfies risk equity preference in the weak sense. This is generally true, with either a zero or non-zero bequest function, as long as $u(.)$ and $v(.)$ satisfy the standard conditions. Note also that, with a logarithmic $g(.)$ function and a zero bequest function, ex ante prioritarianism satisfies risk equity in the *strong* sense. However, this latter result does not extend beyond this special case (see Appendix).

PROPOSITION IV: CBA (using equivalent variations) and ex ante prioritarianism satisfy risk equity preference in the weak sense. CBA (using compensating variations), utilitarianism and ex post prioritarianism do not. Ex ante prioritarianism satisfies risk equity preference in the strong sense under restrictive assumptions regarding *g*(.) and individual utility.

E. Catastrophe Aversion

Keeney (1980) offers a definition of catastrophe aversion which is cited with some frequency in the literature. Assume that policy *a* has a probability π_d of *d* premature deaths and a probability (1- π_d) of no deaths, while policy *b* has a probability $\pi_{d'}$ of d' premature deaths and a probability (1- $\pi_{d'}$) of no deaths. Assume, further, that the two policies have the same number of expected deaths ($d\pi_d = d'\pi_{d'}$), but *d* is less than d'. Then a policymaking tool is catastrophe-averse in Keeney's sense (for short, "Keeney catastrophe averse") if it prefers policy *a* to *b*.

The concept of a mean-preserving spread (Rothschild and Stiglitz 1970) suggests a natural generalization of Keeney catastrophe aversion. Let *D* be a random variable representing the number of fatalities. Let us say that a policymaking tool is "globally catastrophe averse" if it dislikes a mean-preserving spread of *D*. Note that Keeney catastrophe aversion is a particular case of global catastrophe aversion in which *D* is binary with one outcome having zero fatalities.

Definition 5: Let *D′* be a mean-preserving spread of *D*, both random variables. Consider a policy a leading to *D* fatalities and a policy b leading to *D′* fatalities. A social ranking exhibits *strong global catastrophe aversion* iff $a \succ b$ and *weak global catastrophe aversion* iff $a \succ b$ holds whenever all individuals have equal wealth.

Strikingly, not only CBA, but all three of the benchmark SWFs discussed in this Part *fail* catastrophe aversion in both the global sense and the Keeney sense. Indeed, these tools are not catastrophe averse in these senses even if all individuals have the same wealth.

The reason that CBA and the three benchmarks fail these catastrophe-aversion conditions is straightforward. Both CBA and the three benchmark SWFs under consideration assign social value to policies in a manner that is additively separable across individuals. In other words, they all take the form $W(a) = \sum_i f(p_i^O, c_i^O, p_i^a, c_i^a)$.[17] Whether a given individual happens to die in a state where many other individuals do, or in a state where few others do, has no influence on social value.

PROPOSITION V: CBA and the three benchmarks lack the properties of Keeney and of weak global catastrophe aversion.

F. A Summary

Table 1 summarizes the results of this Part, regarding the five properties of interest and whether they are satisfied by VSL and by the three benchmark SWFs.

[17] In the case of CBA, the calculation of individual *i*'s EV for some policy *a* depends *both* on his risk and wealth characteristics in the baseline outcome *O*, *and* on his risk and wealth characteristics with the policy. Thus *f*(.) is a function of p_i^O and c_i^O as well as p_i^a and c_i^a. In the case of the three benchmark SWFs, the arguments for the *f*(.) function are *just* p_i^a and c_i^a. In either event, these policy-evaluation methodologies are separable across individuals.

Table 1					
	Positive Wealth Sensitivity	*Positive Sensitivity to Baseline Risk*	*Equal Value of Risk Reduction*	*Risk Equity Preference*	*Catastrophe Aversion*
CBA	**Yes**	**Yes**	**No**	**Yes** for equivalent variations (weak risk equity)	**No**
Utilitarian SWF	**Yes**	**No**	**No**	**No**	**No**
Ex Post Prioritarian SWF	Depends on *g*(.), *u*(.) and *v*(.). **Yes** under a zero bequest function.	**No**	**Yes**, with appropriate restrictions on *g*(.), *u*(.) and *v*(.)	**No**	**No**
Ex Ante Prioritarian SWF	Depends on *g*(.), *u*(.) and *v*(.).	**Yes**	**No**	**Yes** (to weak form; satisfies strong risk equity with zero bequest function and *g*(.) = log)	**No**

III. **Equal Value of Risk Reduction: A Further Inquiry**

Our analysis in Part II showed that CBA, utilitarianism, and ex ante prioritarianism do not satisfy equal value of risk reduction, and that ex post prioritarianism does not do so except under restrictive parameter assumptions. In undertaking this analysis, as throughout Part II, we assumed that individual utility satisfied the standard constraints in the one-period model: $u(c) > v(c)$, $u'(c) > v'(c)$, and $u''(c) \leq 0$, $v''(c) \leq 0$.

Many seem to find equal value of risk reduction—the equal valuation of lives, independent of individual characteristics—to be a desirable feature of a policy-evaluation methodology (see Baker et al. 2008 and Somanathan 2006). Indeed, this view is reflected in governmental use of population-average rather than differentiated VSL figures. Also, Fankhauser et al. (1997) and Johansson-Stenman (2000) report that one of the most debated issues related to the socio-economic chapter of the Intergovernmental Panel on Climate Change (IPCC) second assessment report was the fact that a smaller value of life was used for poor countries than for rich countries. Interestingly, they compute the implicit numerical values of individual utility and ex ante prioritarian SWF (assuming power forms) that would lead to equal value of risk reduction for specific income differences between rich and poor countries.

In this Part, we consider two different theoretical avenues for securing equal value of risk reduction: introducing some SWF other than the three benchmarks, or relaxing the standard assumptions about the utility function. Our results are negative in the first case, affirmative in the second. We also address what our analysis means for the use of population average VSL.

A. *Is there a Plausible SWF that Satisfies Equal Value of Risk Reduction with the Standard Utility Model*?

Let us consider, first, how variations in the functional form of the SWF might implicate the "equal value of risk reduction" property—holding fixed, for now, the requirement that $u(c) > v(c)$, $u'(c) > v'(c) \geq 0$, and $u''(c) \leq 0$, $v''(c) \leq 0$. We do not consider the rank-weighted or leximin SWFs, which lie beyond the scope of this article, but instead discuss the ex post *transformed* utilitarian SWF; the ex post *transformed* prioritarian SWF; and an SWF suggested by Baker et al. (2008).

It is straightforward to see that ex post transformed utilitarianism does not satisfy equal value of risk reduction with a zero bequest function. Assume that, in the status quo, individual i has wealth c_i and individual j has wealth c_j, with $c_i > c_j$. State s is such that all individuals are dead; it has probability π_s. One policy saves individual i in state s (thus increasing his probability of survival by $\Delta p = \pi_s$); a second policy saves individual j in state s (thus increasing his survival probability by the same amount). With $h(.)$ the transformation function, the increase in social value from the first policy is $\pi_s h(u(c_i)) - \pi_s h(0)$; the increase in social value from the second policy is $\pi_s h(u(c_j)) - \pi_s h(0)$. For these to be the same amount, it must be the case that $h(u(c_i)) = h(u(c_j))$ even though $c_i > c_j$. But $u(c_i) > u(c_j)$, since $u'(.) > 0$, and thus $h(u(c_i)) > h(u(c_j))$ since $h(.)$ is strictly increasing.

In the Appendix, we generalize the proof, showing that ex post transformed utilitarianism does not satisfy equal value of risk reduction with a nonzero bequest function. We also show that ex post transformed prioritarianism does not satisfy this condition.

Baker et al. (2008) suggest one may achieve equal value of risk reduction via *weighted* utilitarianism. (In discussing this proposal, for the sake of clarity, we use superscripts to denote the status quo or alternative policies, so that p_i^O means i's survival probability in the status quo, O, c_j^a j's wealth with policy a, and so forth.)

Let β_i be a weighting factor for individual i equaling $1/(\frac{\partial U_i}{\partial p_i})|_{U_i^O, p_i^O} = 1/(u(c_i^O) - v(c_i^O))$.

Then consider a weighted utilitarian SWF which says that $W(a) = \sum_i \beta_i U_i^a$. This SWF satisfies equal value of risk reduction. If i has baseline survival probability p_i^O and baseline wealth c_i^O, while j has a possibly different baseline survival probability p_j^O and possibly different wealth c_j^O, $\frac{\partial W / \partial p_i}{\partial W / \partial p_j}$ in the baseline is just $\frac{\beta_i (u(c_i^O) - v(c_i^O))}{\beta_j (u(c_j^O) - v(c_j^O))} = \frac{1/(u(c_i^O) - v(c_i^O))}{1/(u(c_j^O) - v(c_j^O))} \frac{(u(c_i^O) - v(c_i^O))}{(u(c_j^O) - v(c_j^O))} = 1$.

However, closer inspection suggests that this SWF is problematic. The most natural interpretation of the Baker et al. (2008) proposal is that the weights are assigned to each individual depending upon her baseline characteristics in O, but are then held "rigid": in order to calculate the sum of weighted utilities for any policy a, the weighting factor for individual i is β_i, regardless of i's characteristics (wealth and survival probability) in a. *This* approach violates the "anonymity" or "impartiality" axiom – a basic principle that any minimally plausible SWF should satisfy. Assume that, in policy a, individuals have wealth and survival probabilities $((c_1, p_1), (c_2, p_2), \dots (c_N, p_N))$, while in policy b these pairs are permuted. Then "anonymity"/"impartiality" requires that a SWF be indifferent between a and b; but the form of weighted utilitarianism now under discussion need not be. [18]

A different interpretation is the weights are not "rigid," but instead assigned by a weighting function. In other words, $W(a) = \sum_i \beta(p_i^a, c_i^a) U_i^a$, where $\beta(p_i^a, c_i^a) = 1/[u(c_i^a) - v(c_i^a)]$. This SWF can violate the Pareto principle (at least if the bequest function is zero). Consider a policy that departs from baseline by increasing some individuals' wealth (by a small or large amount), without changing anyone's survival probability. Then the Pareto principle obviously requires that the policy be preferred, but the SWF now being discussed will be indifferent between policy and baseline.[19]

B. Relaxing Standard Assumptions Regarding the Utility Function

Although Baker et al. focus on the (implausible) weighted-utilitarian SWF, they suggest in a footnote that equal value of risk reduction might also be achieved in an alternative manner—via a utility function such that individual expected utility is separable in wealth and survival probability and linear in the latter. Such an expected utility function violates the standard assumption $u'(c) > v'(c)$. The Baker et al. suggestion therefore invites a more general framing of

[18] It might be protested that failures of anonymity require "large" rather than small departures from the baseline—and Baker et al. (2008) are only proposing their SWF for small changes, see their discussion—but this is not true. Imagine that, in the baseline, one individual has wealth c and another wealth c^*, which is slightly larger, and that they have the same survival probability. Imagine that the policy increases the first individual's wealth to c^* and decreases the second's to c. Then anonymity requires that this "small" departure from the baseline be ranked equally good as baseline; but the "rigid" form of weighted utilitarianism will not do that.

[19] Admittedly, the ex post prioritarian SWF can also violate the Pareto principle. However, such violation only occurs when the social planner is choosing under conditions of uncertainty. By contrast, the weighted-utilitarian SWF under discussion in this paragraph can violate the Pareto principle even if the planner knows, for certain, how individuals will be affected. (Even if p_i^a is one or zero for all individuals and actions, a violation of the Pareto principle can occur.) Arguably, an SWF which conflicts with the Pareto principle under conditions of certainty is *especially* problematic. See generally Adler (2012), Chapter 7, discussing the Pareto principle under certainty and uncertainty.

the question: What are the possible relaxations of the standard utility model that will achieve equal value of risk reduction?

Consider first the possibility of setting $u(c) = v(c) + k$, $k > 0$, $u'(c) = v'(c) > 0$, $u''(c) = v''(c) \leq 0$. It should be stressed that these assumptions are perfectly consistent with expected utility theory. Nor do they seem absurd. If c is defined as wealth *after* insurance premiums and payouts, $u'(.)$ and $v'(.)$ might plausibly be equal, since optimal insurance equalizes the marginal utility of money across states of the world. Inspection of the formulas earlier for $\frac{\partial W}{\partial p_i}$ for W^{CBA}, W^U, W^{EPP}, and W^{EAP} show that, in this case, W^U ("utilitarianism" or, more precisely, ex post untransformed utilitarianism) will secure equal value of risk reduction: $\frac{\partial W^U / \partial p_i}{\partial W^U / \partial p_j}$ will be unity regardless of the survival probability and wealth of i and j. This is *not* true of CBA or of the other two "benchmark" SWFs.[20]

Consider a more dramatic possibility, setting $u(c) = v(c)$ for all c. In this case, both CBA and all of the benchmark SWFs satisfy equal value of risk reduction. But a change to an individual's survival probability has no social value. This is an absurd case, not worth serious discussion.

C. *Population Average VSL figures and equal value of risk reduction*

The analysis of equal value of risk reduction places in a new light the standard governmental practice of using population-average VSL values. Such a practice is problematic, from the perspective of CBA, because VSL is heterogeneous. However, the use of population-average VSL values is *also* problematic from the perspective of *any* SWF that fails equal value of risk reduction. Recall that, with the standard utility model, the utilitarian SWF (even with a transformation function) and ex ante prioritarian SWF do not satisfy equal value of risk reduction, and that the ex post prioritarian SWF (even with a transformation function) does so only under restrictive parameter assumptions regarding $g(.)$, $u(.)$ and $v(.)$.

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[20] Assume that for all c, $u(c) - v(c) = k > 0$, and thus $u'(c) = v'(c)$. Consider two individuals with survival probabilities p_i and p_j and wealth c_i and c_j. Then $\frac{VSL_i}{VSL_j} = \frac{u'(c_j)}{u'(c_i)}$, which is not unity with $c_i \neq c_j$ if $u(.)$ is strictly concave (rather than linear). $\frac{\partial W^{EPP} / \partial p_i}{\partial W^{EPP} / \partial p_j} = \frac{g(u(c_i)) - g(u(c_i) - k)}{g(u(c_j)) - g(u(c_j) - k)}$, which is not unity with $c_i \neq c_j$, since $g(.)$ is strictly concave. Finally, $\frac{\partial W^{EAP} / \partial p_i}{\partial W^{EAP} / \partial p_j} = \frac{g'(u(c_i) - (1 - p_i)k)}{g'(u(c_j) - (1 - p_j)k)}$, which is not unity with $p_i = p_j$ and $c_i \neq c_j$ given the strict concavity of $g(.)$.

For the sake of illustration, consider the utilitarian (ex post untransformed utilitarian) SWF.[21] Imagine that, in the baseline policy, the average VSL value is K. Assume that we have a series of policies, a, b, …, each of which changes individual wealth and survival probabilities from the baseline by a small amount. Policy a corresponds to $(\Delta c_1^a, \ldots, \Delta c_N^a, \Delta p_1^a, \ldots, \Delta p_N^a)$, and so on . For simplicity, assume that each individual's wealth change is the same for all policies: $\Delta c_i^a = \Delta c_i^b$ for all a, b. Then CBA using population-average VSL ranks the policies using the following simple rule: a better than b iff $K(\sum_i \Delta p_i^a) > K(\sum_i \Delta p_i^b)$. For small wealth and probability changes, the utilitarian SWF ranks the policies using the rule: a better than b iff $\sum_i (\Delta p_i^a)(u(c_i) - v(c_i)) > \sum_i (\Delta p_i^b)(u(c_i) - v(c_i))$. Unless (1) all affected individuals have the same wealth in the baseline, or (2) we relax the standard utility model (so that $u(c) - v(c)$ is a constant for all c), the ranking of the policies achieved by the ex post utilitarian rule will *not* generally be the same as the ranking achieved using population-average VSL values.

IV. **Catastrophe Aversion: A Further Inquiry**

We earlier introduced Keeney catastrophe aversion and a more general concept—global catastrophe aversion—and observed that neither CBA, nor the three benchmark SWFs, are globally or Keeney catastrophe averse even if all individuals have the same wealth.

In this Part, we consider the catastrophe-aversion properties of the ex post transformed utilitarian SWF and ex post transformed prioritarian SWF. We continue to focus on catastrophe aversion in the weak sense: where all individuals have the same wealth. Henceforth "weak" will be implicit.

A striking fact is that ex post *transformed* utilitarianism and ex post *transformed* prioritarianism will *satisfy* Keeney catastrophe aversion if and only if the social transformation function $h(.)$ is strictly concave. To see this, consider a population of N individuals out of which d individuals will die if a catastrophe occurs. All have the same wealth c; let "u" and "v" denote $u(c)$ and $v(c)$, respectively. Keep the expected number of deaths n constant, so that the probability of catastrophe is $\pi = n/d$.

Consider ex post transformed utilitarianism. If N-d individuals are alive, the social value of that state, according to ex post transformed utilitarianism, is $h((N-d)u+dv)$. Accordingly, social welfare is equal to $W(d) = (n/d)h(Nu+d(v-u)) + (1-(n/d))h(Nu)$. Keeney catastrophe aversion means that social welfare must be decreasing in the number of fatalities d. That is, there is Keeney catastrophe aversion if and only if $W'(d) < 0$. We easily obtain $W'(d) = -(n/d^2)(h(Nu+d(v-u))-h(Nu)) + (n/d)(v-u)h'(Nu+d(v-u))$. It is straightforward then that $W'(d)$ is negative for all parameters N,u,v and d if and only if $(h(s)-h(r))/(s-r)<h'(r)$ for all s and r such that $s>r$, which indeed holds iff $h(.)$ is strictly concave.

[21] A similar analysis could be provided for any SWF that violates equal value of risk reduction.

It is easy to generalize this result to global catastrophe aversion. If the random number of fatalities is D, social welfare under ex post transformed utilitarianism becomes simply $Eh((N-D)u+Dv)$ in which E is the expectation operator over D. It is immediate then that there is global catastrophe aversion if and only if $h((N-d)u+dv)$ is strictly concave in d, that is, if and only if h is strictly concave.

A parallel analysis shows that ex post transformed prioritarianism satisfies Keeney and global catastrophe aversion iff the transformation function is concave. Continuing the discussion of the previous paragraph: social welfare under ex post transformed *prioritarianism* becomes $Eh((N\text{-}d)g(u)+dg(v))$. There is global catastrophe aversion if and only if $h((N\text{-}d)g(u)+dg(v))$ is concave in d that is, if and only if h is concave.[22]

PROPOSITION VI: Ex post transformed utilitarianism and prioritarianism satisfy Keeney and weak global catastrophe aversion iff the transformation function $h(.)$ is strictly concave.

One immediate implication of this result is that Fleurbaey's (2010) EDE transformation function $h^{EDE}(.)$, combined with utilitarianism or prioritarianism, fails Keeney and global catastrophe aversion.[23] As discussed in Part I, if the underlying SWF is utilitarian, $h^{EDE}(.)$ is linear; if the underlying SWF is prioritarian, $h^{EDE}(.)$ is strictly *convex*.[24]

We can also generalize the incompatibility that Keeney (1980) observes between catastrophe aversion and risk equity. Ex post transformed utilitarianism and prioritarianism do not satisfy risk equity.[25] Thus there is no transformation function that will render ex post utilitarianism, or ex post prioritarianism, consistent with *both* Keeney or global catastrophe aversion and risk equity.

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[22] Although ex post transformed utilitarianism and prioritarianism satisfy weak catastrophe aversion with an appropriate transformation function, they are not necessarily catastrophe averse when individuals can vary in their wealth.

[23] Fleurbaey informally discusses Keeney catastrophe aversion, and suggests that it may make more sense to reduce an independent risk than a risk that hits everyone equally. The intuition is that, when the number of expected fatalities is given, one may prefer a catastrophe with a higher number of fatalities since this reduces ex post inequality. At the limit, if everyone will be either alive or dead, there is maximal ex post equality.

[24] As noted in Part I, where $w = \sum_{i=1}^{N} g(u_i)$, $h^{EDE}(w) = g^{-1}(w/N)$. With $g(.)$ strictly concave, $g^{-1}(.)$ is strictly convex. If $g^{-1}(.)$ is strictly convex in w/N, it is also strictly convex in w.

[25] Focusing on the case where individuals have the same wealth, let u be utility if alive and v if dead. Assume that there are two individuals and three states. In state 1, only individual 1 is alive; in state 2, only individual 2 is alive; in state 3, neither is alive. The status quo probability of state 1 is p_1, and the status quo probability of state 2 is p_2, with $p_2 > p_1$. Assume that a policy increases the probability of state 1 by Δp and decreases the probability of state 2 by the same amount, with $p_1 + \Delta p < p_2 - \Delta p$. Then risk equity requires that the policy be preferred. However, note that the ex post transformed utilitarian SWF assigns the status quo the value $p_1h(u + v) + p_2h(u + v) + (1 - p_1 - p_2)h(2v)$. And it assigns the policy the same value, in violation of risk equity. Moreover, risk equity is violated regardless of the convexity or concavity of $h(.)$. The very same example shows that the ex post transformed prioritarian SWF does not satisfy risk equity.

More generally, some of the policy evaluation tools considered in this article (CBA, the benchmark SWFs, and the ex post transformed utilitarian or prioritarian SWF) satisfy risk equity, some satisfy Keeney and global catastrophe aversion, but none satisfy both.

PROPOSITION VII: Neither CBA, nor utilitarianism or prioritarianism (applied ex ante, or applied ex post with or without a transformation function), satisfy both catastrophe aversion and risk equity.

The failure of CBA to *jointly* satisfy these properties is, thus, a substantially more general feature of policy-evaluation tools.

V. **Conclusion**

Cost-benefit analysis (CBA) evaluates the social gain from reductions in mortality risk using the concept of the value per statistical life (VSL). As a guide to public policy, CBA using VSL exhibits several properties concerning the social value of reducing mortality risk to different people that some commentators perceive to be undesirable, such as positive sensitivity to wealth and unequal value of risk reduction.

We evaluate several prominent social welfare functions (SWFs) and find that these do not necessarily share the same properties as CBA. In particular, the utilitarian SWF, like CBA, exhibits positive wealth sensitivity; the ex post and ex ante prioritarian SWFs may or may not, depending on parametric assumptions. Positive sensitivity to baseline risk (the dead-anyway effect) is characteristic of CBA and the ex ante prioritarian SWF, but the utilitarian and ex post prioritarian SWFs are insensitive to baseline risk. None of the approaches value risk reductions equally in a population, except for the ex post prioritarian SWF under restrictive conditions. Both CBA and the ex ante prioritarian SWF exhibit some preference for risk equity. Neither CBA nor any of the three SWFs exhibit catastrophe aversion, although this property is satisfied by transformed versions of ex post utilitarianism and prioritarianism for appropriate transformation functions when individuals have equal wealth. We strengthen Keeney's (1980) result concerning the impossibility of finding a policy-evaluation methodology that simultaneously satisfies risk equity and catastrophe aversion.

In summary, some combinations of properties that one may find desirable in a method for ranking social policies can be satisfied by appropriate choice of a social welfare function, but others cannot (at least by the SWFs we have examined here). A preference for risk equity requires positive sensitivity to baseline risk and is incompatible with catastrophe aversion. It is interesting to note that the set of properties characteristic of CBA is not inherent to any of the SWFs we have considered, although it can be mimicked by ex ante prioritarianism with appropriate parametric assumptions. Finally, we remind the reader that we have not analyzed how the rank-weighted and leximin SWFs evaluate the social value of risk reduction; that is a topic for future research.

Appendix

A. Comparing Degrees of Wealth-Sensitivity

Consider the case in which $p_i = p_j = p$ and $c_i > c_j$. The article (Part II.A.) characterized CBA and the benchmark SWFs as positively "wealth sensitive" in valuing individual risk reduction if $\frac{\partial W / \partial p_i}{\partial W / \partial p_j} > 1$ in this case. However, we can also compare the *degree* of wealth sensitivity of different policy-evaluation tools. Consider the four ratios, corresponding to CBA and the three benchmarks. Assume the standard utility model.

Ratio 1 (CBA): $$\frac{VSL_i}{VSL_j} = \frac{u(c_i) - v(c_i)}{u(c_j) - v(c_j)} \frac{pu'(c_j) + (1-p)v'(c_j)}{pu'(c_i) + (1-p)v'(c_i)}$$

Ratio 2 (utilitarianism): $$\frac{\partial W^U / \partial p_i}{\partial W^U / \partial p_j} = \frac{u(c_i) - v(c_i)}{u(c_j) - v(c_j)}$$

Ratio 3 (ex post prioritarianism): $$\frac{\partial W^{EPP} / \partial p_i}{\partial W^{EPP} / \partial p_j} = \frac{g(u(c_i)) - g(v(c_i))}{g(u(c_j)) - g(v(c_j))}$$

Ratio 4 (ex ante prioritarianism): $$\frac{\partial W^{EAP} / \partial p_i}{\partial W^{EAP} / \partial p_j} = \frac{u(c_i) - v(c_i)}{u(c_j) - v(c_j)} \frac{g'(pu(c_i) + (1-p)v(c_i))}{g'(pu(c_j) + (1-p)u(c_j)}$$

Note that Ratio 1 and Ratio 2 are greater than unity: both CBA and utilitarian have positive wealth sensitivity. But note also that Ratio 1 is greater than Ratio 2 (as long as $u(.)$ or $v(.)$ or both are concave, an assumption of our standard VSL model). Thus CBA is *more* sensitive to wealth, in valuing risk reduction, than the utilitarian SWF. A given proportional increase in wealth (by c_i/c_j) produces a greater proportional increase in the social value of risk reduction, using CBA as the measure of social value, than if the utilitarian SWF is used as the measure of social value.

Note that this comparison of ratios is an *ordinal* feature of CBA and the utilitarian SWF. If W^{CBA} is replaced by $f(W^{CBA})$ and W^U by $f^*(W^U)$, both f and f^* strictly increasing, then Ratio 1 remains greater than Ratio 2.

Not only is CBA more wealth sensitive than utilitarianism. It is also more wealth sensitive than ex ante prioritarianism: Ratio 1 exceeds Ratio 4. Observe indeed that Ratio 2 exceeds Ratio 4 and thus the result holds since Ratio 1 exceeds Ratio 2.

Finally, CBA is more wealth sensitive than ex post prioritarianism with a zero bequest function or a non-zero bequest function and suitable restrictions on the form of $g(.)$ and $u(.)$. (For instance, $v(.) = ku(.)$, with $0 < k < 1$ and $g(x) = \frac{x^{1-\gamma}}{1-\gamma}$ for any $\gamma > 0$, $\gamma \neq 1$, or $g(x) = \log x$. Under these restrictions, observe that Ratio 2 exceeds Ratio 3 and thus the result holds since Ratio 1 exceeds Ratio 2.)

B. *Risk Equity*

1. <u>CBA with Equivalent Variations Satisfies a Weak Preference for Risk Equity</u>

Individual i has survival probability p_i, individual j has survival probability p_j, with $p_j < p_i$. Both individuals have the same wealth c. If a policy decreases i's survival probability by Δp and increases j's by the same amount, then the individuals' equivalent variations for the policy are as follows, with $\Delta c_i < 0$ and $\Delta c_j > 0$.

(1) $u(c + \Delta c_i)p_i + v(c + \Delta c_i)(1 - p_i) = u(c)(p_i - \Delta p) + v(c)(1 - p_i + \Delta p)$
(2) $u(c + \Delta c_j)p_j + v(c + \Delta c_j)(1 - p_j) = u(c)(p_j + \Delta p) + v(c)(1 - p_j - \Delta p)$

Equation (1) simplifies to:

(3) $[u(c) - u(c + \Delta c_i)]p_i + [v(c) - v(c + \Delta c_i)](1 - p_i) = [u(c) - v(c)]\,\Delta p$

Similarly, (2) simplifies to:

(4) $[u(c + \Delta c_j) - u(c)]p_j + [v(c + \Delta c_j) - v(c)](1 - p_j) = [u(c) - v(c)]\,\Delta p$

Thus:

(5) $$[u(c) - u(c + \Delta c_i)]p_i + [v(c) - v(c + \Delta c_i)](1 - p_i) =$$
$$[u(c + \Delta c_j) - u(c)]p_j + [v(c + \Delta c_j) - v(c)](1 - p_j)$$

Use the abbreviations A^* to mean $[u(c) - u(c + \Delta c_i)]$, B^* to mean $[v(c) - v(c + \Delta c_i)]$, A to mean $[u(c + \Delta c_j) - u(c)]$ and B to mean $[v(c + \Delta c_j) - v(c)]$.

Because $u' > v'$, $A^* > B^*$ and therefore $p_iA^* + (1 - p_i)B^* > p_jA^* + (1 - p_j)B^*$.

It is therefore impossible that $-\Delta c_i = \Delta c_j$. If that were the case, we would have a contradiction. It would follow (given the weak concavity of $u(.)$ and $v(.)$) that $A^* \geq A$ and $B^* \geq B$, and thus that $p_i A^* + (1 - p_i)B^* > p_jA + (1 - p_j)B$, i.e., the left side of equation 5 would be greater than the right. Note, finally, that the term $p_iA^* + (1 - p_i)B^*$, the left side of equation 5, is *decreasing* in Δc_i. (This can be seen by differentiating that term with respect to Δc_i.) Thus, for equation (5) to hold, it must be that $-\Delta c_i < \Delta c_j$, or the sum of equivalent variations is positive and risk equity preference holds.

2. <u>CBA with Compensating Variations Can Violate a Weak Preference for Risk Equity</u>

As before, let individual i have survival probability p_i, and individual j survival probability p_j, with $p_j < p_i$. Both individuals have the same wealth c. If a policy decreases i's survival probability by Δp and increases j's by the same amount, then the individuals' *compensating* variations for the policy are as follows, with $\Delta c_i < 0$ and $\Delta c_j > 0$.

$$(1^*) \qquad u(c)p_i + v(c)(1-p_i) = u(c-\Delta c_i)(p_i-\Delta p) + v(c-\Delta c_i)(1-p_i+\Delta p)$$

$$(2^*) \qquad u(c)p_j + v(c)(1-p_j) = u(c-\Delta c_j)(p_j+\Delta p) + v(c-\Delta c_j)(1-p_j-\Delta p)$$

To see a simple case where $-\Delta c_i > \Delta c_j$ and thus weak risk equity preference fails, let $v(.) = 0$, $p_i = 1$, and $p_j = 0$, and $u(.)$ be the square root function. Equation (1*) simplifies to:

$$(3^*) \qquad c\frac{1-(1-\Delta p)^2}{(1-\Delta p)^2} = -\Delta c_i$$

Equation (2*) simplifies to $\Delta c_j = c$. A little manipulation of (3*) shows that, if $\Delta p > 1-\sqrt{1/2} \approx .3$, then $-\Delta c_i > c$.

3. <u>Ex Ante Prioritarianism</u>

In Part II.D we stated that ex ante prioritarianism satisfies a weak preference for risk equity. This can be easily demonstrated. Assume, as before, $p_i > p_j$ and both individuals have the same wealth c. Assume policy a decreases i's survival probability by Δp and increases j's by Δp, where $p_j + \Delta p \le p_i - \Delta p$. Let U_i^a denote i's expected utility for the policy, i.e., $(p_i - \Delta p)\, u(c) + (1 - p_i + \Delta p)\, v(c)$. Similarly, $U_j^a = (p_j + \Delta p)\, u(c) + (1 - p_j - \Delta p)\, v(c)$. According to ex ante prioritarianism, the change in social value associated with the policy is $g(U_i^a) + g(U_j^a) - g(U_i) - g(U_j)$, so the policy is preferred iff $g(U_j^a) - g(U_j) > g(U_i) - g(U_i^a)$. Note, now, that $U_j^a - U_j = U_i - U_i^a = \Delta p\,[u(c) - v(c)]$, which is greater than zero because $u(c) > v(c)$. Moreover, because $u(c) > v(c)$ and $p_j + \Delta p \le p_i - \Delta p$, it follows that $U_j^a \le U_i^a$. Thus, by strict concavity of $g(.)$, $g(U_j^a) - g(U_j) > g(U_i) - g(U_i^a)$.

In Part II.D., we also indicated that ex ante prioritarianism with a logarithmic $g(.)$ function and a zero bequest function satisfies a strong preference for risk equity (i.e., even where the individuals don't have the same wealth). Indeed, we then have $g(U_j^a) - g(U_j) - g(U_i) + g(U_i^a) = \log(p_j + \Delta p) - \log p_j + \log(p_i - \Delta p) - \log p_i$, which is always positive as soon as $p_j + \Delta p \le p_i - \Delta p$. Nevertheless the result that ex ante prioritarianism satisfies risk equity in the strong sense does not extend beyond the special logarithmic case. Indeed, with a zero bequest function, the logarithmic function is the only strictly concave $g(.)$ function with this property. To see that, observe that wealth has no effect on $g(U_j^a) - g(U_j)$ for an infinitesimal Δp only when $F(c) = g'(pu(c))u(c)$ is independent from c. We obtain $F'(c) = g''(pu(c))pu'(c)u(c) + g'(pu(c))u'(c)$, so that $F'(c) = 0$ for all c is equivalent to $-xg''(x)/g'(x) = 1$ for all x, or $g(.)=\log$.

C. *Equal Value of Risk Reduction*

In Part III.A., we proved that the ex post transformed utilitarian SWF does not satisfy equal value of risk reduction with a zero bequest function. We also indicated that this result generalizes to the case of a non-zero bequest function, and that the ex post transformed prioritarian SWF does not satisfy equal value of risk reduction. We now demonstrate these latter results.

1. <u>Ex Post Transformed Utilitarian SWF</u>

Assume that, in the status quo, individuals i and j are dead in both state s and state s^*. Individual i has wealth c_i and individual j wealth c_j. Both states have the same probability π. Assume that the $N - 2$ individuals other than i, in state s, have total utility L. (In other words, $L = \sum_{k \neq i,j} [l_k^s u(c_k) + (1 - l_k^s) v(c_k)]$, where l_k^s takes the value 1 if individual k is alive in state s and 0 if she is dead, and c_k is k's wealth.) Similarly, assume that the $N - 2$ individuals other than i and j, in state s^*, have total utility L^*.

One policy saves individual i in state s; a second policy saves individual j in state s^*. (Thus the first policy reduces i's fatality risk by π, and the second policy reduces j's fatality risk by the same amount.) The change in social value from the first policy is: $\pi h(L + u(c_i) + v(c_j)) - \pi h(L + v(c_i) + v(c_j))$. The change in social value from the second policy is $\pi h(L^* + v(c_i) + u(c_j)) - \pi h(L^* + v(c_i) + v(c_j))$.

Let $u(.)$ and $v(.)$ be any utility functions that satisfy the standard model. Pick c_j and c_i, L, L^* so that

(1) $c_j < c_i$ and $L + u(c_i) + v(c_j) = L^* + v(c_i) + u(c_j))$.

(Note that it is clearly possible to do this. For example, assume that there are two other individuals k and m, such that $c_k = c_i$ and $c_m = c_j$, and k is alive in s^* and dead in s, while m is dead in s^* and alive in s.)

Because $u' > v'$ by the standard model, and $c_i > c_j$, it follows that $L^* > L$.

In order for ex post transformed utilitarianism to satisfy equal value of risk reduction, $h(.)$, strictly increasing, must be such that

(2) $h(L + u(c_i) + v(c_j)) - h(\mathrm{L} + v(c_i) + v(c_j)) = h(L^* + v(c_i) + u(c_j)) - h(L^* + v(c_i) + v(c_j))$.

Putting (1) and (2) together, plus the fact that $h(.)$ is strictly increasing, it follows that $L = L^*$. But this is impossible, since $L^* > L$.

2. <u>Ex Post Transformed Prioritarian SWF</u>

We ignore the special case where $g(u(c)) - g(v(c))$ is constant, so that ex post prioritarianism without a transformation function satisfies equal value of risk reduction. (See the comment in the footnote before Proposition III.) The question addressed here is whether introducing a transformation function enables ex post prioritarianism to satisfy equal value of risk reduction without these special assumptions on $g(.)$, $u()$ and $v(.)$.

Leaving aside this special case, pick c_i, c_j so that $g(u(c_i)) - g(v(c_i)) \neq g(u(c_j)) - g(v(c_j))$. As above, let states s and s^* have the same probability π, and let both individuals i and j be dead in both states. Let L be $\sum_{k \neq i,j} [l_k^s g(u(c_k)) + (1 - l_k^s) g(v(c_k))]$ and define L^* similarly. Arrange the incomes and alive/dead states of the other individuals so that $L + g(u(c_i)) + g(v(c_j)) = L^* + g(v(c_i)) + g(u(c_j))$. Because $g(u(c_i)) - g(v(c_i)) \neq g(u(c_j)) - g(v(c_j))$, it follows that $L^* \neq L$.

One policy saves individual i in state s, while a second saves individual j in state s^*. In order for equal value of risk reduction to be satisfied, it must be the case that:

$$h(L + g(u(c_i)) + g(v(c_j))) - h(L + g(v(c_i)) + g(v(c_j))) = h(L^* + g(v(c_i)) + g(u(c_j))) - h(L^* + g(v(c_i)) + g(v(c_j)))$$

Because L, L^* have been chosen such that $L + g(u(c_i)) + g(v(c_j)) = L^* + g(v(c_i)) + g(u(c_j))$, it follows from this equation and the fact that $h(.)$ is strictly increasing that $L = L^*$. But this is a contradiction.

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