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Herbert Hovenkamp
University of Pennsylvania Carey Law School

Author ORCID Identifier:
Herbert Hovenkamp 0000-0002-4583-5162

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Arrow’s Theorem: Ordinalism and Republican Government

Herbert Hovenkamp*

I. INTRODUCTION

The theory of social choice tries to identify how individual preferences can be combined into a social policy that meaningfully represents a community’s collective choice. Since collectives themselves cannot choose and we have different ways of aggregating individual preferences, determining a truly “social” choice is often difficult.

Much of the economic literature on social choice has focused on decisionmaking by voters—in particular, by elected representatives in a republican system. In such a system, the electorate generally does not vote directly on policy issues. Rather, it elects representatives to serve it in one or more legislative houses. The electorate controls the representatives’ choices by voting to retain them or to turn them out.

At the intersection of economics and political science, a discipline called “public choice” has grown up to explain the difficulties that elected representatives have in identifying and aggregating the wishes of individual members of the community. Public choice theory addresses such phenomena as republicanism, special interest groups and their lobbyists, legislative bodies and their proceedings, and the interpretation of statutes.

Arrow’s theorem is perhaps the most fearsome tool the public choice theorist owns. True believers can use it to prove, at least to their own satisfaction, that legislative bodies (or groups of voters in general) almost never produce policies that represent the public choice in any meaningful way. This essay examines Arrow’s condition of Independence of Irrelevant Alternatives and argues that it generally fails to obtain in the legislative process. As a result, the other conditions of Arrow’s theorem can obtain at the same time, and legislative voting can yield stable results. Further, to the extent that one is willing to relax certain assumptions of the condition of Independence of Irrelevant Alternatives, legislative outcomes can be said to reflect the welfare needs of constituents in a meaningful, although certainly not perfect, sense. Indeed, the republican system of government contains several mechanisms that prevent the Independence condition from obtain-

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*Ben V. & Dorothy Willie Professor of Law, University of Iowa. B.A. 1969, Calvin College; M.A. 1971, Ph.D. 1976, J.D. 1978, University of Texas. Thanks to Marc Linder and Michael Saks for reading a draft, and to the Nellie Ball Trust for financial support.

1. Among the more pessimistic statements of this position are W. Riker, Liberalism Against Populism: A Confrontation Between the Theory of Democracy and the Theory of Social Choice (1982); Easterbrook, Statutes’ Domains, 50 U. Chi. L. Rev. 533, 547-48 (1983) (“Although legislators have individual lists of desires, priorities, and preferences, it turns out to be difficult, sometimes impossible, to aggregate these lists into a coherent collective choice.”).
These mechanisms work to allow more meaningful and predictable determinations of social welfare than Arrow's theorem would admit.

II. Arrow's Theorem

Although the proofs of Arrow's theorem are quite complicated, the basic concept is not. Arrow specified five minimum conditions for a democratic theory of decisionmaking. The conditions were designed to be relatively uncontroversial—that is, most people should agree that each of the five conditions is necessary to a truly democratic process. Arrow's theorem shows mathematically that not all five conditions can obtain at the same time. By implication, legislative voting processes cannot yield outcomes that are democratic, coherent, and stable.

Arrow specified the following five minimum conditions:

(1) **Unanimity:**
   If one person prefers outcome A over outcome B and all other people either agree with that person or are absolutely indifferent between A and B, then society prefers outcome A over outcome B. This is really a restatement of the Pareto superiority criterion: that a change from the status quo is preferred if at least one person benefits and no one is injured.

(2) **Nondictatorship:**
   No individual has a position such that if he prefers outcome A over outcome B, outcome A will be chosen, no matter what the preferences of others.

(3) **Transitivity:**
   Society's rankings of outcomes must be logically rational in the sense that if society prefers outcome A over outcome B and outcome B over outcome C, it must also prefer outcome A over outcome C.

(4) **Range:**
   The social choice process must consider all relevant individual preferences, and not just some arbitrarily defined subset. For example, if possible choices include outcome A, outcome B, and outcome C, the process may not arbitrarily drop outcome C and force a decision between outcome A and outcome B. Further, all possible preference orderings of A, B, and C are permissible—for example, B, C, A; C, A, B; A, C, B, and so on.

(5) **Independence of Irrelevant Alternatives:**
   With respect to any vote currently on the agenda—for example, as between outcome A and outcome B—how voters would rank a different outcome—for example, C—must be totally irrelevant to the particular vote at hand.

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Arrow's proof shows that if a society contains several voters, three or more options (outcomes) to be voted on, and no unanimity, no process can satisfy these conditions simultaneously. The framework and rationale for Arrow's theorem and its conditions are simple enough. The unanimity assumption identifies Pareto superiority as an acceptable welfare criterion: changes favored by at least one person and opposed by no one should be social improvements. The range assumption simply means that an individual is allowed any preference among all the possible alternatives. The nondictatorship assumption means that no one may intervene to impose her will on others. These assumptions certainly seem important to democratic decisionmaking.

Arrow's theorem may be restated intuitively. Within modern welfare economics, Pareto superiority is about the only acceptable welfare criterion. The proposition of welfare economics that utilities cannot be compared interpersonally makes it impossible for us to look at a government the policy makes policy that benefits some people while it injures others, and proclaim that society as a whole better off. Such transfers are not Pareto superior, for the losers are worse off. Likewise, decisions by majority vote are not Pareto superior moves unless the vote is unanimous. When a move is not unanimously supported, we know that at least one person objects to the policy voted on. Generally, welfare economics does not provide us with a set of tools for concluding that such a policy increases social welfare.

A. Voting Cycles and the Transitivity Condition

Much of the literature relying on Arrow's theorem to critique the legislative process argues that the transitivity condition will fail—that is, there will be an endless cycle of votes and no natural end to the process. Further, the outcomes of such votes are unstable in the sense that they are "path-dependent"—the outcome depends on how the voting agenda is set. Not only might the cycles be unstable, but they are also incoherent in the sense that under one procedure voters will prefer outcome A to outcome B, while under another procedure the same voters with the same preferences and the same set of alternatives will prefer outcome B to outcome A.

For example, suppose that a simple, single-house legislature composed of three legislators—X, Y and Z—is deciding how to spend some government money. The legislators face three choices, which are mutually exclusive: increase crop subsidies ("crop subsidy"), increase welfare payments to the poor ("welfare"), or augment the defense program ("defense"). Assume the three legislators rank their individual preferences in this fashion:

X prefers crop subsidy to welfare, and welfare to defense;
Y prefers welfare to defense, and defense to crop subsidy; and
Z prefers defense to crop subsidy, and crop subsidy to welfare.

Under these circumstances, the outcome of the vote will depend entirely on how the agenda is set. Since different agendas can yield inconsistent outcomes, Arrow's transitivity condition will not be met. For example,
suppose the legislators are asked first to vote between crop subsidy and welfare. X and Z will prefer crop subsidy, while only Y will prefer welfare; so crop subsidy will win. Now the legislators will be asked to vote between crop subsidy and defense. X will vote for crop subsidy, while Y and Z will prefer defense; so defense will win.

But suppose that the legislators are asked first to choose between welfare and defense. In this case X and Y will vote for welfare, while only Z will vote for defense; so welfare will win. The second vote will be between welfare and crop subsidy. Crop subsidy, preferred by X and Z, will win. The first agenda thus yields a social preference of defense over crop subsidy and the second agenda yields a preference of crop subsidy over defense. Therefore, Arrow's transitivity condition has not been met. This phenomenon is known as "cycling." If the controller of the agenda permits the participants to continue voting, it could theoretically go on forever, yielding a repeating pattern of inconsistent outcomes. Increasing the number of possible alternatives or the number of voters only increases the possibility of cycling.

A good deal of the literature on Arrow's theorem and legislative decisionmaking examines the institutional mechanisms that legislative bodies such as Congress use to prevent cycling and create stable outcomes. These include, for example, bicameral legislative structures and committees. Such institutional mechanisms may indeed improve stability. They do not, however, entail that the legislative outcome will be the best approximation of aggregate individual choice.

B. Ordinalism and the Independence Condition

Arrow's Independence condition sounds quite reasonable when it is stated, but there is more than meets the eye. The Independence condition is violated whenever a voter deciding between, say, outcome A or outcome B, bases her choice on how she feels about some additional outcome, C, or on her perception of how some other voter might feel about C. It is also violated when the voter compares the strength of her preference for A over B with the strength of another's preference for B over A. For example, suppose X in the above illustration has a very weak preference for crop subsidies over welfare—she is almost, but not quite, indifferent. On the other hand, X feels extremely strongly that no more money should be spent on defense. She also knows how the other two legislators feel. When the first vote is taken, between crop subsidy and welfare, X votes welfare rather than crop subsidy even though crop subsidy is her first choice. As a result, the second vote will be between welfare and defense, and welfare will win, because both X and Y prefer welfare. X "loses" in the sense that her second choice, not her first, won the vote; but X "wins" in the sense that her least preferred choice was not chosen. In reality, X wins because she was almost indifferent as between her first and second choices, while X really did not


want her last choice at all. Such "strategic" voting violates the Independence condition and tends to yield stable outcomes when a voter refuses to vote her first choice in order to increase the chances of getting her second choice. Actually, such voting is no more "strategic" than a person's market-driven decision to purchase her second choice when she cannot afford her first.

The purpose of Arrow's Independence condition is to define a social choice mechanism that is completely ordinal in its weightings of preferences. "Ordinal" in this sense means that preferences only can be ranked. They can never be measured in some more cardinal fashion, such as "X prefers A to B by 10%, but prefers B to C by 300%." Most importantly, they can never be quantified and compared across two or more individuals. For example, under purely ordinal rankings we can conclude that "both X and Y prefer A to B," but cannot add "however, X's preference is far stronger than Y's." Such quantitative statements are attempts to cardinalize preferences and then make quantitative interpersonal comparisons of them. As such, they violate the Independence condition.

Ordinalism is essential to Arrow's theorem. In fact, if complete information about cardinal utilities were available, determination of social welfare would be quite easy. The state could simply place each unit of every entitlement where it produced the greatest marginal utility, and total utility would be maximized. Identifying the optimal social choice becomes a problem only when utilities cannot be quantified and interpersonally compared. As argued below, however, many mechanisms employed in the legislative process inject a measure of cardinality into welfare assessments. To the extent that welfare measurement becomes cardinal and comparable interpersonally, an essential condition of Arrow's theorem fails to obtain.

Importantly, Arrow's theorem is not a proof that a particular criterion for measuring social welfare is substantively inaccurate or false. Rather, it is a proof that under stated conditions individual, purely ordinal preferences cannot be aggregated into a unique social welfare function. The theorem may simply not apply to alternative mechanisms for measuring welfare.

An example is wealth maximization, the method for measuring welfare defended by Judge Posner, under which optimal welfare is equated

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8. Amartya Sen has shown that intrapersonal cardinal ranking of preferences is consistent with the Independence condition; however, interpersonal cardinality is not. See A. Sen, supra note 9, at 338; A. Sen, Collective Choice and Social Welfare 129-30 (1970) (theorem 8:2).

with maximum social wealth. Since wealth can be measured cardinally by a constant unit (dollars), it is comparable interpersonally. In fact, if we assume a constant marginal utility of wealth, we can aggregate social wealth simply by summing the wealth of each owner. As a result, under utility maximization (and perfect information) an equilibrium social welfare function can result. For example, if legislators operating under the constraint of wealth maximization chose from three alternatives, A, B and C, that increase social wealth by $10, $50, and $100, respectively, they would all vote for C. Not only would the outcome be stable, but Arrow's unanimity condition would be satisfied as well.

One might make numerous criticisms of wealth maximization as a mechanism for determining social welfare, but Arrow's theorem would not be among the critiques. Wealth maximization is only one of many alternative, "cardinal" mechanisms for measuring welfare that are not subject to Arrow's theorem, or that produce sufficient consensus that we can say that the unanimity condition is satisfied. The extent to which Arrow's theorem applies to decisionmaking by legislative bodies in a republican system of government depends largely on whether decisionmaking by such bodies should be viewed as the aggregation of ordinal preferences, as Arrow's theorem would suggest, or whether they are guided by alternative, "cardinal" mechanisms for determining welfare.

III. REPUBLICAN GOVERNMENT AND CARDINALITY PRODUCING MECHANISMS

Whether the Independence condition applies in the legislative process thus depends on one's basic conception of the process itself. If one views the process as a scheme in which a group of people assert naked preferences without regard to the preferences of others, the condition seems to apply. Arrow wished to take into account the common assumption of modern welfare economics that interpersonal comparisons of cardinal utilities are impossible. Under that assumption, for example, X might state a categorical preference for vanilla ice cream over chocolate, while Y states equally categorically that chocolate is better than vanilla. But these statements do not permit us to conclude that X likes vanilla ice cream more than Y likes chocolate. The statements do not even permit us to conclude that X likes vanilla ice cream more than Y likes vanilla. Although Y may like vanilla ice cream much less than chocolate, and X much more, Y may nonetheless like vanilla ice cream more than X likes vanilla ice cream. This may be so because Y likes ice cream in general much more than X; because X hates all ice cream, and simply hates chocolate more; or because Y has much stronger taste perceptions and obtains far more pleasure from food than X does. Unfortunately, these subjective feelings are never capable of being measured unless they are entirely our own.

11. Wealth-maximizing policies have both gainers and losers, and thus would not receive unanimous support if each voter voted her own wealth. See J. Murphy & J. Coleman, Philosophy of Law 222-27 (rev. ed. 1990). However, elected representatives who agreed to vote for the wealth-maximizing choice and acted in good faith should approve that choice unanimously, provided that they all agreed about which choice was in fact wealth maximizing.
12. On the historical development of the problem of interpersonal incomparability, see
In an important sense, the republican theory of public decisionmaking by elected representatives seems quite inconsistent with the premises of Arrow's theorem. Arrow's theorem applies to ordinally ranked individual preferences. It treats legislators something like children selecting a single flavor of ice cream to be shared by all. Each child's preferences are strictly individual, and there is generally no reason to prefer the preferences of one child over those of another. Likewise, the children do not take the strengths of one another's preferences into account.

A more optimistic view of legislative decisionmaking, sometimes termed Madisonian republicanism in its strongest version, believes that legislators have both the obligation and capacity to identify the public interest. Within the republican view, legislators both discern a public interest and educate their constituencies about it. In discerning the public interest, Madisonian legislators necessarily make objective, rather than subjective, assessments about welfare. To the extent this can be done in a systematic or scientific fashion, consensus emerges, and consensus creates equilibrium.

But an optimistic Madisonian view of republican democracy is not necessary to convince one that the legislative process is capable of working better than Arrow's theorem suggests. Much weaker versions are sufficient to undermine the application of Arrow's theorem to legislative process. In fact, cardinality and interpersonal comparability of individual welfare functions are all but inherent in the process of representative decisionmaking. To the extent that the Independence condition is not satisfied, Arrow's theorem does not show a failure of stable outcome.

Each process listed below describes a mechanism of legislative decision in which the strict requirement of ordinality (i.e., Arrow's Independence condition) fails to obtain. Collectively, they might be referred to as "cardinality-producing processes." In some cardinality-producing processes, Arrow's condition of nondictatorship also may not be satisfied. Some of these cardinality-producing processes may also yield unanimity, and thus satisfy the one condition Arrow recognized as producing a stable social welfare function.

One's sense of the pervasiveness of the application of Arrow's theorem to the legislative process depends largely on whether one believes that cardinality-producing processes dominate legislative decisionmaking, or that legislative decisions are purely ordinal—based entirely on unranked, uncomparable preferences of individual decisionmakers, like children picking ice cream. My list of cardinality-producing mechanisms, which may not be exhaustive, includes the following:

A. Shared ideological perceptions of the national interest;
B. Compelling scientific or theoretical models;

C. Objective welfare judgments;
D. Cost-benefit analysis;
E. Log-rolling;
F. Risk assessment and cycling avoidance; and
G. Multi-district lobbying or financial support.

I will discuss each of these in turn.

A. Shared Ideological Perceptions of the National Interest

The most optimistically Madisonian notion of republicanism has a group of legislators who share certain fundamental values but who differ about the details of state policy. The legislator's job is both to pass good legislation—legislation that maximizes social welfare consistent with these shared values—and to educate his constituency about what the legislature is doing and why. Such a view may account for some legislative outcomes. For example, it may explain why a consensus of Congress votes for civil rights legislation or for reparations to Japanese internment victims even though individual "aye" voters believe that the legislation will make their particular districts economically worse off—in fact, even if a majority of the legislator's constituents oppose such measures.

The ongoing debate between defenders of the Madisonian vision and public choice theory boils down to a dispute about how pervasively this shared vision governs legislative outcomes. Even the most optimistic Madisonian would undoubtedly concede that a good deal of legislation seems inconsistent with any notion of a shared set of fundamental democratic values. But, the most pessimistic public choice theorist must also concede that shared perceptions of the public interest sometimes triumph over all opposition. Many recent empirical studies of legislative behavior have found that: (1) legislative outcomes are much more predictable and stable than Arrow's theorem suggests;\textsuperscript{14} and (2) basic political and ethical ideology remains the single best predictor of legislative votes.\textsuperscript{15} Values still count, and shared values are calculated to produce consensus.

B. The Compelling Model

Perhaps the greatest builder of consensus in the modern world is the scientific model, whether it be neoclassicism, Newtonian physics, relativity, or Darwinism. Models are theories about how some aspect of the world works. Implicit in models are ideas about how the world can be made to work better. The best models are themselves simple, at least in their basic


outline. Yet they are also able to explain large amounts of information, and thus provide meaningful answers to the questions that society currently asks.

Consider a group of astrophysicists, engineers, and other scientists planning a complex voyage to Mars, an experiment not attempted before and about which there is some uncertainty. Each scientist involved in, say, planning the trajectory has a set of ideas about what is best. Assume that the scientists ultimately make their decision by majority vote. Would we expect cycling, or a final choice that bears only an arbitrary, haphazard relationship to the correct trajectory? Certainly, the scientists could be wrong. But it is not obvious that the decision made by the several scientists is less accurate than a decision made by one scientist. In fact, we may have good reason to think the opposite: that models produced by scientific consensus are more accurate than those produced by a sole scientist. Scientists using the scientific method share in a common body of information and employ a model that produces consensus. The scientists also collectively agree that a successful mission is better than an unsuccessful one.

Models become successful to the extent that they explain better than their alternatives. As a model's predictions meet success, the model tends to attract consensus, perhaps first in a narrowly defined technical community, but eventually in a larger part of society. For example, American enterprise law in the nineteenth century was heavily driven by concepts developed within classical political economy, one of the most powerful models in human history. The law changed drastically when classicism and neoclassicism gave way to later economic models that displayed less confidence in the unregulated market. 16

The biological model of environmentalism, which replaced strict genetic determinism in the 1930s and 1940s, is another such model. Originally the model fought for acceptance among natural scientists. Then it spread to social scientists and to other communities. The Supreme Court desegregation decisions and the Civil Rights movement can in part be explained as the triumph of environmentalism over geneticism as a theory about race and educability. 17 Likewise, the so-called deregulation movement in federal legislation had its origin in the Democratic Carter administration, but developed most fully during the Reagan administration. That movement was driven heavily by a new economic model that displaced older theories about regulation, and suggested that the role of potential competition was much broader than it had been before. 18

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The classical theory of the corporation, the great regulatory movements inspired by the New Deal, the judicial and legislative civil rights movement, and the deregulation movement are all products to one degree or another of the scientific revolutions that preceded them. The body of judicial decisions that tells the legislature whether a particular choice violates the Constitution is an important example of such a model; when legislators make objective judgments about the constitutionality of proposed legislation and avoid voting for unconstitutional proposals, the range of legislative preferences is restricted.

The impact of a body of common legal rules on a group of constrained voters can be dramatic. For example, Judge Easterbrook has attempted to apply Arrow's theorem to the Supreme Court. He suggests that the Court is incapable of making consistent decisions because Arrow's theorem predicts cycling and irrationality in voting patterns. If the ranking of the Justices' preferences was purely random, he might have a point. For example, if there were as few as four possible outcomes for any particular Supreme Court decision and preferences were randomly distributed, we would expect unanimity in only one out of 65,000 cases. If there were as few as two outcomes (plaintiff/prosecutor wins; defendant wins) we would expect unanimity only once in 250 cases. In fact, the Supreme Court manages to make unanimous decisions between one-third and one-half of the time. Even for an economist, 20,000 times more occurrences than a model predicts is not a very robust prediction.

To the extent that these models give the political community a set of shared answers to certain problems, they inject an element of cardinality into related welfare judgments. This is not to say that any particular model is "right" or "wrong" in some absolute sense, but rather that while the model reigns, a consensus emerges that a particular public problem has a particular solution.


20. Very few Supreme Court cases would have as few as two-possible outcomes, and most would have more than four. A two outcome case would occur when certification was limited to a single issue and there were only two permissible ways the case could be decided. If certiorari is granted on two issues, and either has two possible outcomes, then there are four possible distributions of judicial votes.

The formula for predicting unanimity when preferences are distributed at random is:

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P = \frac{N!}{N^x},
\]

where \( P \) = the probability of unanimity;
\( N \) = the number of options to be voted on; and
\( x \) = the number of voters.

21. I selected at random two volumes of the Supreme Court Reporter, 95 and 105. In volume 95 if a unanimous opinion with a concurrence is counted as unanimous, 50.4% of the Supreme Court's decisions were unanimous; if a unanimous decision with a concurrence is counted as non-unanimous, 40% are unanimous. In volume 105, if a unanimous opinion with a concurrence is counted as unanimous, 49.2% of the Supreme Court's decisions were unanimous; if a unanimous decision with a concurrence is counted as non-unanimous, 31% are unanimous. The mere counting of decisions unanimous as to outcome but with concurring opinions as non-unanimous entails that there are at least three options. In that case, one decision in 19,683 would be unanimous if preferences were randomly distributed.
C. **Objective Welfare Judgments**

Both ordinalism as an economic principle and positivism as a principle of scientific fact finding insist that human well-being cannot be measured by the external observer. One person can never experience another's sense of satisfaction or happiness. All such measurements must be conducted through surrogates. In the case of economics, the most commonly used surrogate is revealed preference, or observations of people's decisions to purchase in economic markets. Other social sciences and medicine place far more confidence in more "objective" welfare judgments, such as housing quality, health, education, employment or stated satisfaction.

The objective welfare judgment is the most ubiquitous cardinality-producing devise in legislative determination of social welfare. Economists as a group tend to disparage objective welfare judgments, because their methodology conceives of "value" as based only on subjective choice. But psychologists, sociologists, educators, political scientists, public health officials, regulators, transportation engineers, and many other specialists—as well as lay persons—make objective welfare judgments all the time. Even economists, particularly macroeconomists involved in policy matters, find such judgments unavoidable. In fact, wealth-maximization, the theory that welfare is maximized when wealth is maximized, can purport to be a theory about welfare only if it rests on an objective judgment that there is a close correlation between wealth and subjective well-being. That correlation cannot be observed directly in others, although by introspection we might be able to find it in ourselves.

Congress continually makes objective welfare judgments. In setting a federal speed limit, legislative committees look at accident statistics, not at the preferences of those on the highway. When Congress decides on a federal drug-use policy, its members do not spend much time considering the market-based preferences of drug dealers or users or even of those associated with drug users. Some data, such as information about the effect of drug use on family members or other associates of the user, may not show up in markets at all—at least not in any useable form. Instead of looking at market data, Congress looks at objective data about health effects, employment effects, family effects (such as divorce rates or instances of child abuse), and so on. It presumes that people with good health have more welfare than people with poor health, or that children from whole families have more welfare than children from broken ones. Likewise, the members of Congress who voted to place health warning labels on cigarette packages likely did not count the votes of their constituency; nor did they respond to some perceived market choice. Very likely, they were moved by an array of reports that found cigarette smoking to be harmful to health, and they made an objective judgment that bad health decreases welfare.

Every public official must continually make judgments about what makes people better or worse off without reliable information derived from the purely market choices of her constituents. First of all, the electoral process does not provide information about all issues, unless they are very prominent in a campaign. Second, even where markets exist, we do not always have good information about the nature and extent of preferences.
Third, the objects of legislative policy often are incapable of being traded on a market, or may be traded in such a camouflaged fashion that subjective value is difficult to assess. Even when considering explicitly "economic" legislation such as the minimum wage, legislators look at things like projected unemployment statistics—once again making an objective welfare judgment that people earning the minimum wage would prefer to be employed. The more reliable, robust, and unambiguous objective information is, the more calculated it is to produce consensus.

Incidentally, the objective welfare judgment is not the exclusive vice of legislative bodies. All public decisionmakers, whether the benevolent dictator or the common-law judge, must rely on it to one degree or another. Even the market must be regulated by someone with authority, and the decisions will almost certainly be informed to some extent by objective judgments about what is best for the relevant community.

The objective welfare judgment, just as the strongly held set of common values and the consensus building model, reflects the degree to which the legislative process is educative rather than merely reflective. The job of elected representatives in the republican system is not merely to do what the constituents want; it is also to give the constituents what is good for them—sometimes whether they want it or not. Arrow's theorem entirely overlooks this paternalistic, educational function of the legislative process. At any given time a voting majority of the electorate might oppose tight money, strictly enforced speed laws, school taxes, or public utility rate increases. While people often take the short view of things, the legislator's job is to take the long view.

D. Cost-benefit Analysis

One need not be an optimistic Madisonian in order to find cardinality-producing devices that govern a good deal of legislative decisionmaking. Many of the most pessimistic would accept the role of cost-benefit analysis in producing stable legislative results.

As noted above, Arrow's theorem applies only if preferences are strictly ordinal and incomparable from one person to another. If preferences could be ranked by a unit that presumptively applied equally to all participants, then they could be ordered cardinaly and compared across persons. Cost-benefit analysis produces cardinality by the simple device of translating all preferences into market values, measured in a constant currency such as dollars.

To be sure, cost-benefit analysis supplies only an imperfect surrogate for subjective welfare. It does not measure people's subjective preferences as such, but only the aggregate value of their preferences as reflected in the market. Further, built into cost-benefit analysis are other assumptions that seem contrary to our experience—such as the assumption that the marginal utility of income is constant, and that different people obtain equal amounts of welfare from a dollar.22 The important issue here, however, is

22. On these points, see Hovenkamp, Legislation, Well-Being, and Public Choice, supra note 5, at 68-74.
not that the welfare measure produced by cost-benefit analysis is accurate, but that it is *shared*. To the extent that a group of decisionmakers rely on cost-benefit analysis to analyze a particular legislative proposal, they tend to arrive at consensus about the optimal course of action. To that extent, stable legislative solutions can be expected to emerge.

Wealth maximization, the theory of social choice put forward by Judge Richard Posner, is merely a broad application of a kind of cost-benefit analysis. Posner's commitment to wealth maximization probably explains why he has not been much of a supporter of those parts of public choice theory drawn from Arrow's theorem. For Posner, legislative failure to achieve the public interest occurs when legislators fail to enact the wealth-maximizing solution because they heed the call of a particular special interest group. But Posner clearly believes that stable legislative outcomes are possible, provided that the wealth-maximizing solution can be identified and that the legislators honor their ethical duty to vote accordingly.

Whatever one might think of the merits of wealth maximization, it is a device for injecting cardinality into the measurement of social welfare. To the extent that the policy maker can assume, as wealth maximization does, that dollars and units of welfare are equivalent, the social welfare function becomes nothing more than the maximization of total wealth.

To be sure, not all legislative policies based strictly on dollar values will yield determinate outcomes. Even cost-benefit analysis will not prevent cycling when purely distributional issues are concerned. For example, suppose that $100 is to be divided among A, B, and C by majority vote. A proposes that A and B each take $50 and C take nothing, and the proposal wins when A and B vote for it. C then will propose to A that A receive $51 and C $49; since A will be better off, C and A will agree. B then will propose that C receive $50 and B $50, and B and C will agree; and so on, and so on. This is a division game that has no "core," or equilibrium outcome, and produces endless cycles. Not only legislative redistribution games, but also markets, can fail to have a core. The theory of the core, therefore, is not simply a critique of legislative decisionmaking; it can operate as a critique of both market and legislative decisionmaking.

But the theory of the core yields a failure of equilibrium in the legislative process only when legislators are playing a zero-sum game—for example, when they are dividing up a sum of money that everyone wants and the way the money is divided has no impact on total social wealth.

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26. On the core, see L. Telser, Economic Theory and the Core 3-4 (1978). In Arrow's words, "For any allocation which gives some individual, say 1, a positive amount, there is another, which gives 1 nothing and divides up his share in the first allocation among all the others; the second is preferred to the first by all but one individual." K.J. Arrow, Tullock and An Existence Theorem, in 1 Collected Papers of Kenneth J. Arrow 87 (1983).
Cost-benefit analysis will yield such failures only in the unusual case where all measured alternatives receive exactly the same score. For example, if a legislature is contemplating project alternatives X, Y, and Z and all three alternatives produce precisely the same net benefit, then legislators would be forced to rely on criteria other than cost-benefit analysis, such as individual preference, in order to rank their choices. Such outcomes should be quite rare. In the more usual case—where Y produces greater net benefits than either X or Z—any legislator constrained to apply cost-benefit analysis must vote for Y.

Finally, even purely distributional questions have stable solutions if we can make interpersonal comparisons of the utility of income across different individuals. For example, if we arrive at a consensus that the poor as a group obtain more utility from a given amount of money than do the wealthy, then a group of legislators who agreed in that judgment would be inclined to support wealth transfers from wealthy to poor. The same group of legislators would be less inclined to support wealth transfers from the poor to wealthy, or simply within the class of wealthy.

The much grosser set of judgments that legislators make when comparing the benefits and burdens of an outcome in their particular district with the benefits and costs in another is closely related to cost-benefit analysis. For example, suppose that proposed legislation would be of enormous benefit to four out of ten legislative districts, but would impose very small costs on the other six districts. Will the representatives from the six districts that will have costs imposed upon them oppose the legislation? Even in the absence of technical cost-benefit analysis, legislators make assessments of economic losses and gains, and they do not necessarily support every trivial gain to their own district no matter what the perceived costs to other districts. This phenomenon is inconsistent with Arrow's Independence condition. When it occurs, a measure of cardinality emerges, tending to produce consensus.

E. Logrolling

The most reliable producer of outcomes that reflect social choices is the market. Even those who are most pessimistic about the ability of legislative bodies to discover the public's choice retain a high degree of confidence in markets. Markets tend to assign goods or other entitlements to those who value them most highly, and total social welfare is presumptively maximized when everything is held by the person who values it the most.

Logrolling, or the trading of votes among legislators, can invest the most purely ordinal of preferences with a strong element of cardinality. For example, assume three legislators—X, Y, and Z—and two sets of policy alternatives under consideration—A, B, and C, and P, Q, and R. Suppose the preferences of the legislators are as follows:

X: A, B, C; P, Q, R
Y: B, A, C; Q, P, R
Z: C, A, B; R, P, Q.
If each set of proposals is considered separately, a failure of equilibrium (cyclical voting) could obtain. But suppose that X approaches Y and offers to vote for B provided that Y will vote for P. The result will be that B will win on the first issue and P will win on the second. Since in both cases a majority of voters (X and Y) now have the same ordinal ranking, this outcome is stable, or immune from cycling.\(^{28}\)

This trade will take place if: (1) X's preference of P over Q is stronger than X's preference of A over B; and (2) Y's preference of Q over P is weaker than Y's preference of B over A. Under these circumstances, the trading of votes becomes a substitute for the otherwise impossible ordinal comparison of preferences. The “market” for votes has assigned them where their values are the highest.

Logrolling tends to stabilize voting by creating consensus about those interests on which the voters place the greatest value.\(^ {29}\) In this case, however, “value” is not necessarily monetary value, and the results of logrolling cannot be expected to mimic the results of cost-benefit analysis or wealth maximization. Legislators may have certain preferences for ideological reasons, or may have constituents who have such ideological preferences. There is no principled reason ideological preferences cannot be traded as much as economic preferences. Though a libertarian may favor deregulation of insurance and oppose a prohibition on flag burning, if she is a good politician she might be willing to trade the former away in order to protect the latter, which she considers more important. Once again, logrolling tends to produce stability.\(^ {30}\)

F. Risk Assessment and Cycling Avoidance

Any model of legislation that predicts endless voting cycles and no outcome at all must assume that the cost of endless cycles is near zero. But a majority of voters presumably want some outcome. In fact, if a majority wanted no outcome at all, then “no outcome” would be an equilibrium solution. Those who have relatively weak preferences for their first choices over their second choices will be more willing to exchange their first choice for a stable outcome. The result is a tradeoff phenomenon somewhat akin to logrolling except that no exchange necessarily takes place between two different representatives. Rather, some representatives who can foresee an endless cycle reverse their voting preferences in order to create an equilibrium. This phenomenon violates Arrow’s Independence condition, because it means that a voter will take the preferences of others into consideration in an effort to stop the cycle. For example, coalitions may form among people who agree about their first and second choices,

\(^{28}\) See supra text accompanying note 5.


\(^{30}\) Logrolling may result in excessive public expenditure, under certain conditions. See generally Tullock, Some Problems of Majority Voting, 67 J. Pol. Econ. 571 (1959). For a critique of this position, see D. Mueller, supra note 2, at 75.
although not about their ranking. The result is that equilibria can be created, and members of the coalition are guaranteed their second choice or better.

Arrow's theorem relies on the rather odd behavioral assumption that participants in a voting process doggedly rank their own subjective preferences but never even consider the preferences of others. The Independence requirement demands such an assumption. But an important element in any theory of utility or value is risk, which Arrow's theorem completely ignores. To a person who is risk neutral and experiences constant marginal utility of income, a 50% chance of receiving $1,000,000 is worth only $500,000; in all events, it is worth less than $1,000,000.

Even if utilities are absolutely incomparable, one will consider the known or anticipated preferences of others if these reduce the chances that one's own preferences will be realized. To return again to the earlier example:

X prefers crop subsidy to welfare, and welfare to defense;
Y prefers welfare to defense, and defense to crop subsidy; and
Z prefers defense to crop subsidy, and crop subsidy to welfare.

Suppose that X has a weak preference for crop subsidy over welfare, but a strong preference for welfare over defense. Suppose he could cardinalize his own preferences as crop subsidy 6, welfare 5, and defense 1. X also has a pretty good idea how Y and Z will vote. In this case the expected utility of crop subsidy to X is not 6, but rather 6 discounted by the likelihood that crop subsidy will be selected. If X's voting for crop subsidy results in endless cycling and no selection of anything, the utility of voting for crop subsidy is zero. More likely, if X's voting for crop subsidy results in an arbitrary breaking of the cycle at an unknown point, the utility of a vote for crop subsidy is 2, or 6 multiplied by a one-third probability that the cycle will stop at crop subsidy rather than welfare or defense. On the other hand, if X votes welfare, and X knows that Y is voting welfare, he will receive a cycle-proof majority. So the expected value of welfare to X is 5. The risk adjusted utility values for X are really something like crop subsidy 2, welfare 5, defense 1. Coalition formation proceeds on such reasoning.

Several observations should be made about this method of determining preference rankings. First, the extent to which X takes the preferences of other voters into account in calculating the utility of his own preferences depends on the size of the voting body. If the voting body were the national electorate, the preferences of another voter might have an immeasurably small effect on the outcome. But in the typical legislative body such considerations are quite important, particularly in close votes. For example, a United States Senator might easily be expected to vote for her second choice in order to avoid some other choice that she regards as far less desirable. This is an important part of the legislature's compromise and coalition formation process, and does not require logrolling.

Second, the theory that a person measures the utility of his own preferences by taking the preferences of others into account does not depend on perfect information about the preferences of others. It could depend on fairly rough statistical or even anecdotal information, such as party affiliation or prior voting records or prior statements. The informa-
tion must merely be sufficient to indicate to a representative or coalition that it has a much better risk of seeing its second choice realized than its first; when adjusted by this risk, the second choice has higher utility.

Third, the notion that voters consider the preferences of others in this fashion does not require the interpersonal comparison of utilities; it requires only that the voter make a purely subjective judgment of the value of a certain policy to her and the risk that the policy will or will not be voted through. All information about other voters is strictly about their ordinal rankings of preferences.

Fourth, such voting is not strategic in any reasonable sense of that term. In fact, once a vote is adjusted for risk, X is merely voting for the choice that gives X the greatest utility. This is no more "strategic" than a person’s decision not to purchase a $1.00 lottery ticket for a one in a million chance to win a $10,000 car. The utility of the dollar is greater than the utility of the car, discounted by the risk of not winning it.

Finally, one must remember that Arrow’s theorem is a mathematical theorem, not an empirical observation. It can be proven only if its conditions are assumed. To the extent that a voter takes the preference orderings of others into account in determining his own vote, Arrow’s condition of Independence of Irrelevant Alternatives is violated. Thus for any choice process in which at least one voter makes utility-risk calculations as described here, Arrow’s theorem has no relevance.

G. Multi-district31 Lobbying or Financial Support

One of the more pessimistic elements of public choice is the theory of special interest groups. The theory suggests that inefficient legislation is often passed because small, single-minded special interests are much more effective at communicating their wishes to legislators than are large groups that represent diverse interests.32

Both Arrow’s theorem and the theory of special interest groups may explain why legislation is sometimes not in the public interest. But the two theories are fundamentally inconsistent with each other. Arrow’s theorem suggests that when all the given conditions obtain, legislative results are indeterminate. The theory of special interest groups is based on the notion that certain groups, because of their structure and homogeneity, do a particularly good job of obtaining the legislation they want, or of defeating the legislation that they do not want. To be successful, the lobbyist must work in more than one district. The goal of any lobbyist is to unify the preferences of enough legislators that the lobbyist’s goal will be achieved. Successful lobbying should increase rather than decrease the amount of

31. “District” is used here to describe the geographic area that elects a particular representative. It could be a state, as in the case of a United States senator, or an area smaller than a county, as in the case of a state assembly member or township commissioner.

determinacy in legislation, although the resulting legislation may not be any more in the public interest than legislation passed in the complete absence of lobbying.

IV. How Good Must Welfare Information Be?

Some of the legal literature on legislation and Arrow's theorem suggests that the theorem establishes conclusively that legislative bodies cannot achieve equilibrium outcomes. For example, Jonathan Macey talks of the "logical impossibility" of consistency. On the other hand, most empirical studies suggest that cycling and observable inconsistency occur only rarely in actual deliberative bodies. The cardinality-producing devices discussed in this essay tend to produce both equilibria and consistency. But how often do they achieve this result?

Surprisingly, the preconditions for stability and consistency are quite weak. It can quite easily be shown that if a majority of voters has the same first choice among any given set of alternatives, that first choice will win in any majority-rule process for selecting a single alternative, no matter how the voters rank the other choices. Likewise, if a majority of voters has exactly the same ordinal ranking of preferences (no cardinality is necessary), it can easily be shown that this particular ranking will emerge in any voting scheme in which the majority rules. In short, a cardinality-producing scheme must be sufficiently strong to produce a consensus among a majority as to the first choice any time the first choice is all that is important. That choice will then be immune from cycling. If a ranking of choices is important—presumably a rare thing in legislative process—then a majority with respect to that ranking will produce the


35. For example, assume a legislative body of n voters asked to choose among P alternatives (A, B, C, D ... P), and that a majority ((1/2)n + 1) prefer alternative A as their first choice. In that case, for any other alternative in the set, say, X, not more than (1/2)n−1 of the voters can claim it as first choice. In the strongest possible case against A, suppose that X is the first choice of (1/2)n−1 voters, and the second choice of the (1/2)n + 1 voters for whom A is the first choice. In that case, in a process of voting by opposing pairs and eliminating losers, X will win in any pair in which A is not included, but X will lose when finally pitted against A. Likewise, in a full-slate initial vote, plus run-off if necessary, A will obtain (1/2)n + 1 votes on the first ballot, and no alternative could obtain more than (1/2)n−1; therefore, A would win without a run-off.

36. See Greenberg, Consistent Majority Rules Over Compact Sets of Alternatives, 47 Econometrica 627 (1979); Kramer, On a Class of Equilibrium Conditions for Majority Rule, 41 Econometrica 285 (Mar. 1973); see also Buchanan, Individual Choice in Voting and the Market, 62 J. Pol. Econ. 334 (1954). Likewise, Gordon Tullock has shown that the more cohesiveness exists among voters, the less often cycling will occur. See G. Tullock, Toward A Mathematics of Politics ch. 3 (1967). In fact, he concludes "that when a rather simple and probable type of interdependence is assumed among the preference functions of the choosing individuals . . . , 'Arrow problems' will seldom be of much importance." Id. at 37.

37. Rare, but not unheard of. For example, suppose that a city council, rather than the electorate, elects the mayor under a scheme in which the member receiving the most votes becomes mayor, while the member receiving the second most votes becomes vice mayor. All
Further, although it is logically possible that voting on three or more alternatives will yield indeterminate cycles when there is no majority agreement on the ranking, the probability of such cycles is usually quite low. In general, the likelihood of a cycle increases with the number of voters and the number of alternatives under consideration. The probability of cycles can be computed if we assume that preferences are randomly distributed among the voters. The assumption of randomness naturally gives us the highest probability of a cycle; to the extent that the cardinality-producing devices discussed earlier produce any amount of consensus, preferences will not be random among the voters but will tend to line up behind some alternatives more than others. As the Supreme Court example discussed earlier suggests, any amount of consensus can greatly reduce the likelihood of cycling. The assumption of randomness may overstate the probability of cycling by hundreds or even thousands of times.

Even assuming complete randomness of preferences, it can be shown that cycling among a large number of voters faced with three alternatives will occur less than ten percent of the time. If there are as many as five alternatives, cycling will occur about one-quarter of the time, assuming that preferences are distributed absolutely at random. Only when there are more than ten alternatives does cycling become more likely than not. These probabilities are based on the assumption of a “naked” voting system with no devices such as bicameralism, agenda-control, or threat of presidential veto to break the cycle. Further, they are computed on the premise that the cost of cycling is zero.

For example, suppose that the council has five members with the following preferences:

<table>
<thead>
<tr>
<th>Member:</th>
<th>A B C D E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranking:</td>
<td></td>
</tr>
<tr>
<td>A A A E E</td>
<td></td>
</tr>
<tr>
<td>B B B C D</td>
<td></td>
</tr>
<tr>
<td>C D E A B</td>
<td></td>
</tr>
<tr>
<td>D E C D A</td>
<td></td>
</tr>
<tr>
<td>E C D B C</td>
<td></td>
</tr>
</tbody>
</table>

If the voters simply voted once for their first choice, under a rule that gave mayor to the person with the most votes and vice mayor to the person with the second most votes, A would be mayor and E vice mayor. But this would violate the majority rule condition, because E did not obtain a majority. In a second ballot, from which A’s name (as winner on the first ballot) was removed, B would win.

In any order of pairwise voting, A would prevail over all competitors in the first round. In the second round, when A’s name was removed, B would prevail against all competitors. As the illustration suggests, it is not necessary that a majority of voters have identical preference rankings from top to bottom; they must merely be identical for as many ranks as the relevant decisional rule requires them to produce.

V. MARKETS AND COMPARATIVE ADVANTAGE

An important ideological subtext of some of the work using Arrow's theorem as a critique of legislative process is that markets are better than legislatures at identifying the social welfare. An important premise of welfare economics is that perfect markets will yield trades until a Pareto optimal position is reached—that is, until no further transfer could make one person better off without making someone else worse off. Unfortunately, there are an infinite number of Pareto optimal positions. Different initial entitlements will yield Pareto optimal outcomes, but the outcomes will not be the same. If in one society everyone had exactly the same initial endowment of resources and in another society one person had all the resources and the others none, free trade in both societies would eventuate in a Pareto optimal distribution of resources. But the market cannot discriminate among (i.e., rank) these Pareto optimal outcomes—that is, it has no basis for showing that one Pareto optimal outcome yields more social welfare than another unless it is willing to indulge in interpersonal comparisons of utilities. If cardinal utilities cannot be interpersonally compared, no market-based system of allocating resources that permits an unrestricted range of initial endowments can be shown to yield a discrete social welfare function.

A. Market Cycling

Any belief that economic markets have the comparative advantage is undermined by the fact that market allocations can be just as prone to instability as democratic allocations. Market transactions occur when the participants have an opportunity to produce an economic surplus—that is, when the seller places less monetary value on the thing traded than the buyer does. Economic analysis of markets and their participants can tell us a great deal about whether transactions will occur, but it generally says little about how the economic surplus will be divided. This is particularly true when the market is not perfectly competitive, as it is in most of the interesting situations in which we apply the Coase theorem to analyze legal rules. Much of the Coase theorem literature glosses over the question of how the participants to the bargaining process will divide up any economic surplus that a transaction produces. Division of the surplus is a zero-sum game in economic markets just as it is in political ones. Further, it generates identical cycling problems.

Consider Coase's simplest example of the farmer and the neighboring cattle raiser whose cattle damage the farmer's crops. Suppose that the farmer values the entitlement to be free of the damage at $100 and the

39. See Easterbrook, supra note 19.
41. In the second society there would not be any exchanges, for the have-nots would have nothing to offer. Since no reallocation could make the have-nots better off without making the single have worse off, the society would have reached a Pareto-optimal state.
42. For an illustration, see J. Bonner, supra note 9, at 111-30.
cattle raiser values the entitlement of not having to fence at $150. The Coase theorem says that the parties will strike a bargain; if the farmer has a trespass action, the cattle raiser will pay the farmer between $100 and $150 for a right that the cattle trespass on the farmer's land and damage the crops.

But the amount of the payment could be $101, $149, or something in between. Coase's theorem says nothing about how to determine the payment's size, but only that profit-maximizing actors will strike a bargain and this bargain, once reached, will be efficient. That is, the outcome will be Pareto optimal even though an alternative bargain (one with a larger or smaller payment from cattle raiser to farmer) would have been Pareto optimal as well.

Why do the parties not engage in an endless round of negotiation as to how the entitlement will be divided? If transacting were literally costless in every sense of the word, perhaps they would. But the Coase theorem does not generally acknowledge endless transacting as a possibility. Many writers about bilateral monopoly argue that such markets often perform poorly and the costs of negotiating are rather high. But they do not generally conclude that the parties will engage in endless bargaining, even though the principle—dividing up a sum of money in a way that has no effect upon the efficiency of the outcome—is precisely the same as division of the surplus among voters.

The similarities between the wealth transfer problem in the Coasean and the legislative situations become more obvious when the transaction involves many participants. Good examples are zoning and air quality regulation, where a single decision might affect hundreds of people. For example, suppose operation of a cement factory is worth $100 to its owners, and imposes $7 in losses on each of ten homeowners. The factory's operation is actionable by any one of the ten homeowners. Now, the factory must bargain with the homeowners, obtain a release from all of them, and be willing to pay them off to make continued operation of the factory possible. There is a $30 surplus to be divided. Presumably the factory and each homeowner will attempt to capture as large a portion of that surplus as possible.

If the factory negotiates with each homeowner separately, there will be holdout problems. It can afford to pay some homeowners, say, $12, but only if it pays others less. There is no obvious reason the wrangling could not go on indefinitely. Alternatively, the factory might place, say, $90 in escrow, and tell the ten homeowners to divide it among themselves. Then the problem has become identical to the legislative wealth transfer problem that has no core, with the added complication that each homeowner might also threaten to walk away, making the entire pot worthless. No homeowner will accept less than $7, but if the pot were divided equally each would get $9. Homeowners 1 through 8 might agree with each other that they will let 9 and 10 have only $7.50 each, and keep the remaining $75.00 for

themselves, giving them around $9.35 each. But homeowners 9 and 10 might then respond by: (1) refusing to bargain at all; or (2) offering owners 1 through 6 a deal that will make them a little better off than under the previous deal, but give 7 and 8 only slightly more than $7 and so forth. In short, the chaos problem shows up in exactly the same fashion as it does in the legislative situation. Once again, however, any solution that emerges (other than endless cycling) is Pareto optimal—in the parlance of Coase theorem literature, “efficient.”

With respect to the Coase theorem, it is often said that the absence of transaction costs forces the efficient result. Transaction costs might prevent the efficient outcome from occurring. In the legislative problem the presence of transaction costs seems to be what forces the equilibrium—that is, if cycling is costless it will go on forever. In fact, however, the term “transaction costs” is being used in these two models in slightly different ways. If bargaining were indeed costless in the broadest sense—that is, if it had no time value or opportunity cost—the parties would engage in endless bargaining for division of the surplus in a Coasean economic market just as much as in the legislative market. One situation is as amenable to cycling as another. Further, economics gives us no basis for concluding that any particular division of the surplus is better than another. All yield Pareto optimal equilibria.

The Coasean market might appear to contain a mechanism for stopping cycles that the legislative market does not have: contract law. Once the parties have agreed, they are bound to their agreement. But agreement in a market setting requires unanimity, not just a majority, and there is nothing to suggest that unanimity will ever be obtained.

B. Market Failure and Legislative Equilibrium

One perspective on the proper relationship between economic markets and political markets is that of the classical political economists, who had a great deal of faith in the market and very little in politics. Within their laissez faire model, most resources should be allocated through economic markets. The government should intervene only when markets were not performing properly—that is, in cases of “market failure.” If correction of these failures were in everyone’s best interest, the corrections might be supported by everyone. Within this model, the state was obliged to stay entirely out of the business of transferring wealth, but should provide only those things that the market fails to provide in the proper amount, such as lighthouses, public streets, or national defense.

The provision of national defense, which is close to being a pure public good, illustrates a classic market failure. Suppose that a ten-member society can be defended for $1000, but that for any individual member or subset of members to defend themselves would also cost $1000. Suppose further that if any person or subset purchased $1000 worth of defense, the defense thus

procured would protect the entire society. In this case no one, according to
the classic theory, will purchase defense on her own. Rather, each will rely
on the fact that someone else is doing it, and will free ride on the
expenditures of others.48

But now suppose the government proposes to tax each member of this
society by $100 in order to provide defense. Would the proposal receive
unanimous consent? Perhaps. Each person would have defense, and at far
lower cost than if each had to purchase it alone. If each person valued
defense by at least $100, the proposal would make each better off. Several
neoclassical economists believed the provision of pure public goods would
be supported by unanimous consent.49 In that case, provision of such a
good would satisfy Arrow's unanimity condition and could be said to be in
the public interest. Some economists believe that only unanimity rules can
be justified as efficient political alternatives to private markets.50

Strategic voting might upset unanimity. If I thought most others
wanted defense badly enough, I might threaten to vote "no" unless my
share were reduced and that of others increased. Likewise, if I thought
someone else was going to buy defense no matter how the vote turned out,
I would probably vote no. Inequality of assets to be protected might also
interfere with unanimity. I might value defense at precisely $100, but
suspect that someone else values it at $400 because she has more assets to
be protected. In that case I might vote "no" for two reasons: (1) I might
hold out for a different distribution of the tax, perhaps in proportion to
asset value; or (2) I might assume that the wealthier people would be likely
to purchase defense in any event if the outcome of the vote is negative. We
might wish to address this problem as Knut Wicksell and Erik Lindahl have,
by suggesting a tax on each person in proportion to the value that each
places on defense.51 But such a scheme will almost certainly present
problems of strategic behavior: people will have an incentive to understatedhe
value of defense to themselves.

The public choice literature uses Arrow's theorem as the basis for an
argument that legislative allocations of resources are inherently unstable
and likely to generate endless cycles. However, the argument goes, if the
legislature confined itself to interfering in the market only in the case of
substantial market failure, as in the provision of a pure public good, then
stable outcomes in the public interest could be assured. Such outcomes
should at least theoretically receive unanimous support and be immune
from cycling.

Taxation, in id., at 72. An important critique is P. Samuelson, The Pure Theory of Public Expenditure, 36 Rev. Econ. & Statistics 387 (1954); see also Vickrey, Utility, Strategy, and Social
Decision Rules, 74 Q. J. Econ. 507 (1960). For subsequent developments, see D. Mueller, supra
note 2, at 123-34; Special Supplement, 29 Public Choice 2 (Spring 1977).
50. E.g., J. Buchanan, supra note 47.
51. Lindahl and Wicksell, supra note 49.
But even in this case unanimity—indeed, freedom from cycling—is elusive. In fact, there is no reason for thinking that the legislative provision of pure public goods—or the minimal state's interference with the market only to correct market failure—is any more immune from cycling than are legislative proposals in general.

The problem with many treatments of unanimity and public goods is that they divide the domain of possible legislative solutions into two choices and only two: provision or nonprovision. But legislators do not vote for "some solution" to a public goods problem; they must vote for a particular solution. Further, the Range condition in Arrow's theorem requires that all possible solutions to the public goods problem be entitled to be considered. Suppose now that a society must decide the question of providing a national defense. Further, suppose that there are only four possible alternatives (more than four alternatives only makes the cycling problem more likely):

A) Provide a defense to be paid for by a progressive income tax;
B) Provide a defense to be paid for by a property tax;
C) Provide a defense to be paid for by a head tax; or
D) Do nothing.

The fact that defense is a pure public good means that alternative D is Pareto inferior to any of the other three options. However, that tells us nothing about the rankings of the other options. In general, those with relatively high income but relatively less property may prefer the property tax; those with valuable property but relatively low income may prefer the income tax; those with both high income and property may prefer the head tax, and so on. There could be three individuals in this society, or perhaps three groups of individuals who have these preferences:

X prefers A to B, B to C, and C to D;
Y prefers B to C, C to A, and A to D;
Z prefers C to A, A to B, and B to D.

From this point one can easily see cycling problems similar to those noted earlier. In any system of pairwise voting, D (no defense) will be defeated. But there will be an endless cycle among the remaining three options. If the first pair is between A and B, A will win; then in a round between A and C, C will win; then in a vote between C and B, B will win; and so forth. Not only will unanimity fail, but there will be no stability whatsoever. In short, "unanimity" with respect to public goods problems means only that everyone agrees that doing nothing at all is not the best solution. On the policy decision about what to do, however, disagreement can exist over as wide a range as any other legislative decision.

The implications of this should not be overlooked: Arrow's theorem is not a constraint on "distributive" legislation as opposed to "efficiency" legislation. It does not suggest that pork barrel legislation is prone to cycling while efficient legislation designed to correct market failures will yield stable social welfare functions. Arrow's theorem suggests that all legislative solutions, whether efficient or otherwise, are inherently unstable and cannot yield determinative social welfare functions. This is true even of legislation designed to correct market failure, or legislation designed to facilitate markets in the first place. As a result, one cannot use Arrow's
theorem as a basis for dividing the territory between types of legislation and suggest that legislatures confine themselves to correcting market failures. If all the conditions of Arrow's theorem apply, then the theorem applies to all legislative solutions, not merely to those designed to redistribute wealth. Legislatures should simply go out of business.

VI. Conclusion

Arrow's theorem takes ordinalism as a premise and finds that democratic process can lead to chaos. That result has often been used to indict the democratic process. But which propositions constitute the premise and which the conclusion are entirely a matter of how one drafts the equation. Clearly, it seems, the conditions assumed by Arrow’s theorem and democratic decisionmaking are inconsistent with one another. But one could as easily state the theorem this way:

If one wishes to have a rational (transitive) and democratic (nondictatorial) process for state policymaking, judgments about social welfare must be based on some criterion other than purely ordinal rankings of preferences.

Under the strictest constraints of ordinalism, all individual preferences are treated equally, and the only institution capable of ranking them is the market. That particular assumption of ordinalism—that the intensity of wants or needs cannot be measured and interpersonally compared—actually yields a much greater degree of uncertainty than even Arrow's theorem acknowledges. Even in cases in which Arrow's theorem allows a stable equilibrium—such as when more than half the voters have the same first choice—determination of social welfare is impossible: the 49% who represent the minority may have individual preferences that are far stronger than the 51% who represent the majority.

But the principal task of legislative process is the ranking of preferences. The biggest failure of Arrow's theorem as a critique of the legislative process is that the economist's notion of "welfare" as something that can be measured only through revealed preference in markets is not the only meaning of welfare; certainly, it does not capture the notion of welfare that legislators commonly employ. When more objective, categorical concepts of individual welfare are admitted, determination of social welfare becomes much more meaningful and predictable.

Arrow's theorem should be understood by the legislator as a warning that legislative intent can come in two kinds. First is the intent to pass legislation in the public interest, whether measured by cost-benefit analysis or objective welfare judgments. Second is the intent to give effect to one's own selfish preferences or those of one's constituency. Legislation of the second kind is calculated to fall short of the public interest, and it is more likely to be unstable as well.