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The Reduced Form of Litigation Models and the Plaintiff's Win Rate

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Abstract

In this paper I introduce what I call the reduced form approach to studying the plaintiff's win rate in litigation selection models. A reduced form comprises a joint distribution of plaintiff's and defendant's beliefs concerning the probability that the plaintiff would win in the event a dispute were litigated; a conditional win rate function that tells us the *actual* probability of a plaintiff win in the event of litigation, given the parties' subjective beliefs; and a litigation rule that provides the probability that a case will be litigated given the two parties' beliefs. I show how models with very different-looking structure can be understood in common reduced form terms, and I then use the reduced form to prove several general results. First, a generalized version of the Priest-Klein model can be used to represent any other model's reduced form, even though the Priest-Klein model uses the Landes-Posner-Gould ("LPG") litigation rule while some other models do not. Second, Shavell's famous any-win-rate result holds generally, even in models with party belief distributions that are both highly accurate and identical across plaintiffs and defendants. Third, there are only limited conditions under which the LPG litigation rule can be rejected empirically; this result undermines the case against the LPG rules' admittedly non-optimizing approach to modeling litigation selection. Finally, I use the reduced form approach to clarify how selection effects complicate the use of data on the plaintiff's win rate to measure changes in legal rules. The result, I suggest, is that recent work by Klerman & Lee advocating the use of such data is unduly optimistic.

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1 Introduction

In this paper I introduce the reduced form approach to studying the plaintiff's win rate in litigation models. The core of a reduced form is a pair of plaintiff and defendant beliefs concerning the probability that the plaintiff would win in the event a dispute were litigated. Beyond that, a reduced form involves three elements. First is a function specifying the joint distribution across disputes of the parties' respective subjective beliefs concerning the probability the plaintiff would win in litigation. Second is a conditional win rate function, whose value is the actual probability the plaintiff would win if the case were litigated, given the parties' subjective beliefs.

The third element is a litigation rule, which tells us the probability that a case will be litigated given the two parties' subjective beliefs. Many contemporary litigation models have reduced form litigation rules that are generated as the equilibrium of explicit optimizing behavior by the parties. An alternative litigation rule that has at times been influential is the Landes-Posner-Gould ("LPG") litigation rule that results from assuming cases are settled if and only if the parties would realize surplus from avoiding litigation.¹ The LPG litigation rule is embedded in Priest & Klein's (1984) model, as well as a number of more recent papers. Because the LPG litigation rule is difficult to motivate as the result of optimizing behavior, it fell out of favor. Even so, it has made something of a comeback in recent years,² perhaps due to its simplicity. One mission of this paper is to offer additional reasons why the LPG litigation rule should be taken seriously by litigation model builders.

A major advantage of the reduced form approach is that it allows seemingly disparate models to be discussed in common terms. I show in section 2 how to find the reduced form of: (i) one-sided screening and signaling models developed by Bebchuk (1984), Shavell (1996), and Klerman & Lee (2014), (ii) the two-sided model developed and simulated by Priest & Klein (1984), using the LPG litigation rule, and (iii) Friedman & Wittman's (2007) two-sided model in which settlement is determined via an explicit optimizing framework.³

In section 3, I prove a surprising result: a generalized form of Priest and Klein's model is sufficiently flexible to represent the reduced form of any litigation model. This representation theorem is especially notable because the generalized Priest-Klein model uses the LPG litigation rule to determine litigation/settlement outcomes. Thus, even litigation models in which explicit optimizing behavior determines the litigation/settlement equilibrium share a reduced form with a model that is based on the black boxish LPG litigation rule. The key to this result is allowing the parameters representing parties' stakes and costs of litigation and settlement to vary across cases; this generally ignored variation provides the flexibility necessary for the generalized Priest-Klein model to match arbitrary litigation rules.

¹See Landes (1971), Posner (1973), and Gould (1973).

²See, e.g., Lee & Klerman (2014), Klerman & Lee (2014), Rosenberg & Spier (forthcoming, Part III.4), Prescott & Spier (2016), Gelbach (2015), and Schweizer (2016).

³I shall not attempt a general review of the voluminous literature concerning the general properties of litigation models; several excellent recent literature reviews are Spier (2007), Daughety & Reinganum (2012), and Wickelgren (2013).

section 4 is motivated by Shavell's (1996) seminal paper using a simple screening model to demonstrate, as the title declares, that "any frequency of plaintiff victory at trial is possible." I ask whether the same result holds the LPG litigation rule. In light of section 3's representation theorem, the answer to this question is yes since Shavell's model itself can be represented using a generalized Priest-Klein model. But there is more to learn by considering Shavell's argument, rather than just his result. Shavell was writing in response to Priest & Klein's (1984) suggestion that, among litigated cases, there is a general tendency of the plaintiff's win rate toward 50%.⁴ Shavell suggested that Priest and Klein's 50% tendency was driven by special symmetry and accuracy features of the parties' belief distributions, and his proof that any win rate is possible relies on a one-sided screening model in which these features are absent. But contrary to Shavell (1996), I show that any plaintiff's win rate can be observed among litigated cases even when the parties have symmetric belief distributions, even as these distributions converge to perfect accuracy, and even in a reduced form that has the LPG litigation rule. This section thus shows that the any-win-rate result is broader than has generally been realized. It also implies that the conditions under which there is a tendency toward a 50% plaintiff's win rate among litigated cases are more special than has been realized.⁵

In section 5, I focus on the question of when observable data on litigated cases might be sufficient to reject the hypothesis that the LPG litigation rule holds. The representation theorem in section 3 implies that it will never be possible to reject the LPG litigation rule unless we impose conditions on the distribution of the parties' stakes and cost parameters. It is conceivable that one might find a data set that contains at least some measure of each of these parameters. Accordingly, in this section I ask whether such data, together with the plaintiff's win rate among litigated cases and the share of litigated cases, could ever be sufficient to reject the LPG litigation rule. The answer is yes, which suggests that the litigation rule in effect might be empirically testable. However, when few disputes are litigated—as is empirically true—the conditions for rejecting the LPG litigation rule appear to have limited practical relevance. Thus, there is little empirical basis for rejecting the LPG litigation rule.

In section 6, I turn to the question of whether data on the plaintiff's win rate among litigated cases can be useful to understand the direction of change in legal rules. One of the implications of Priest & Klein (1984) is that selection effects related to settlement may render the plaintiff's win rate among litigated cases uninformative about the state of the law. Klerman & Lee (2014) recently launched an apparently powerful attack on this implication by proving, in several of the same models I consider here, that there exist conditions under which the plaintiff's win rate among litigated cases rises when legal rules change in favor of

⁴A number of other authors have attempted to analyze the properties of the plaintiff's win rate in the presence of settlement-induced selection in litigation. A partial list of notable work includes Wittman (1985); Eisenberg (1990); Hylton (1993); Hylton & Lin (2012) and Friedman & Wittman (2007). For more discussion of this literature, see Lee & Klerman (2015) and Klerman & Lee (2014).

⁵I develop sufficient conditions for the 50% result in an in-process companion paper, Gelbach (2016).

plaintiffs. Consequently, Klerman & Lee “hope[] to open up avenues for empirical research that have previously been neglected as unfruitful and to give legitimacy to those who analyzed plaintiff victory rates in spite of Priest and Kleins arguments.” Page 210.⁶ But using the reduced form approach to understand how selection effects operate following changes in legal rules shows that Klerman & Lee’s sufficient conditions are difficult to evaluate. Models in which their conditions are satisfied are tough to distinguish from those in which they are not, so that assuming Klerman & Lee’s conditions appears little different from simply assuming that selection effects are not problematic in the first place.

Throughout, I shall deliberately abstract from procedural facts that characterize real-world procedure (e.g., multiple claims; multiple parties on each side of the “v”; the motion to dismiss; discovery; summary judgment). This is not because these details are unimportant, nor because my reduced form framework could not be generalized to account these features. These details are simply beyond the scope of this work.

2 The Reduced Form Approach

Let Q_p be the plaintiff’s subjective probability that the plaintiff would win if the case were to be litigated. Similarly, let Q_d be the subjective probability the defendant places on the same event, i.e., that the plaintiff would win. In any case with given stakes and cost parameters, party beliefs are fully characterized by the belief pair (Q_d, Q_p) .

Define \mathcal{L} as an indicator variable that equals 1 when the parties litigate and 0 otherwise. Define \mathcal{W} as an indicator variable that equals 1 if the plaintiff would win were there litigation, and 0 otherwise. We can fully characterize the litigated share of cases and the plaintiff’s win rate among litigated cases using the joint distribution of (Q_d, Q_p, L, W) . Using the law of iterated expectations, the litigated share is $P(\mathcal{L}=1) = E_{Q_d, Q_p}\{E[\mathcal{L}|Q_d=q_d, Q_p=q_p]\}$ (where subscripts on the expectation operator denote the random variables over whose distributions the expectation is taken). The plaintiff’s win rate among litigated cases is $P(\mathcal{W}=1|\mathcal{L}=1) = E_{Q_d, Q_p}\{E[\mathcal{W}|\mathcal{L}=1, Q_d=q_d, Q_p=q_p]\}$. The three functional elements of a reduced form litigation model are:

1. A joint cumulative distribution function of party beliefs, $F_{Q_d Q_p}$, such that $P(Q_d \leq q_d \ \& \ Q_p \leq q_p) = F_{Q_d Q_p}(q_d, q_p)$. This probability distribution may be either discrete, continuous, or a mixture of the two. I refer to the probability density function or probability function, as appropriate, as $F_{Q_d Q_p}$, and I use the term $f_{Q_d Q_p}$ for the associated probability density or mass function, as appropriate.
2. A litigation rule, L , such that $L(q_d, q_p) \equiv E[\mathcal{L}|Q_d=q_d, Q_p=q_p]\}$. The litigation rule tells us the probability that a case will be litigated, given the parties’ subjective beliefs.

⁶For recent examples of studies attempting to study the effects of pleading standard changes due to *Bell Atlantic Corp. v. Twombly*, 550 U.S. 554 (2007) and *Ashcroft v. Iqbal*, 556 U.S. 662 (2009), see, e.g., Moore (2012) and Hubbard (2013). See Gelbach (2011) and Gelbach (2014) for a selection-based critique of this approach.

3. A conditional win rate function, w , such that $w(q_d, q_p) \equiv E_{Q_d Q_p}[\mathcal{W} | \mathcal{L}=1, Q_d=q_d, Q_p=q_p]$. Given the subjective beliefs (q_d, q_p) , the conditional win rate function tells us the probability that the plaintiff would win, in the counterfactual circumstance that the case were to be litigated.

To illustrate the flexibility of the reduced form approach, I now show how it encompasses several important litigation models in the literature.

2.1 One-Sided Asymmetric Information Models

In both the Bebchuk and LK asymmetric information models, one of the two parties knows the true probability that the plaintiff would win in litigation. Therefore the conditional win rate function is

$$w(q_d, q_p) = \begin{cases} q_d, & \text{if defendants are informed,} \\ q_p, & \text{if plaintiffs are informed.} \end{cases} \quad (1)$$

The other party does not know the true probability that the plaintiff would win the dispute, but this uninformed party does know the correct population distribution of this probability. In my terms, the uninformed party's belief concerning the plaintiff's probability of winning is thus the population average value $\bar{q} \equiv \int_0^1 Q f_Q(q) dq$; notice that this means that in these models, *all* uninformed parties have the same subjective belief \bar{q} . Because the conditional win rate function is invariant to one party's beliefs in these models, I say that it is unilateral.

The joint density of beliefs in these models (and the joint probability function when discrete) may be written

$$f_{Q_d Q_p}(q_d, q_p) = \begin{cases} f_Q(q_d), & \text{if } q_p = \bar{q} \text{ and defendants are informed,} \\ f_Q(q_p), & \text{if } q_d = \bar{q} \text{ and plaintiffs are informed,} \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

All models with the above conditional win rate function and joint distribution of party beliefs satisfy a belief-consistency condition I call conditional mean consistency: they all have the property that the average of the conditional win rate function given a single party's belief equals that belief. Thus, $E_{Q_p}[w(q_d, Q_p)] = q_d$, and $E_{Q_d}[w(Q_d, q_p)] = q_p$. In addition, because one party is uninformed in these models, with constant beliefs about the plaintiff's probability of winning, all these models have what I call unilaterally degenerate beliefs.

Where the screening and signaling models part ways is in the litigation rule. In screening models, the uninformed party makes a settlement demand or offer. If the informed party accepts, the case settles, and if the informed party rejects, the case is litigated. In the KL

signaling model, the roles are reversed: the informed party makes a settlement demand or offer, and the uninformed party accepts or rejects.

2.1.1 The Bebchuk screening model

In the Bebchuk screening model, the informed party's beliefs are continuously distributed. For simplicity I shall discuss only the situation in which the defendant is informed; the qualitative results are very similar when the plaintiff is the informed party. If x_p is the plaintiff's settlement demand, the informed defendant will settle whenever the defendant's belief about the plaintiff's probability of winning exceeds the threshold $(x_p - c_d)/J_d$, where c_d is the defendant's cost of litigating and J_d is the defendant's expected cost of losing at trial. Writing x_p^* for the plaintiff's optimum settlement demand, this means that defendants will reject the settlement demand if and only if their belief is less than $q_d^* \equiv (x_p^* - c_d)/J_d$.⁷ Thus we have the litigation rule

$$L(q_d, q_p) = \begin{cases} 1, & \text{if defendants are informed and } q_d < q_d^*, \\ 0, & \text{otherwise.} \end{cases}$$

This litigation rule is binary, since all cases sharing a given belief pair are either litigated for certain or settled for certain. The Bebchuk model's litigation rule is also unilaterally increasing in party optimism. This is so because the litigation probability never falls as the informed party becomes more optimistic, but it sometimes increases.⁸ The litigation rule's relationship to party optimism is unilateral in the Bebchuk model because the uninformed party's belief does not enter the litigation rule. A final property of the litigation rule in the Bebchuk model is that it depends on the joint distribution of party beliefs. This is so because if we change the density of beliefs held by informed defendants, we will generally change the threshold's value, which would then yield a distinct litigation rule. For convenience, Table 1 collects a number of the properties I have discussed in connection with the one-sided asymmetric information models in general, and with the Bebchuk model in particular; it also provides a similar taxonomy for the other models to be discussed below.

In (q_d, q_p) -space, the entire set of litigated cases lies to the left of a vertical litigation frontier.⁹ Because the threshold q_d^* depends on the shape of the belief distribution, the location of the litigation frontier in the Bebchuk model depends on the distribution of party beliefs.

⁷With settlement costs assumed to be zero, the optimal settlement demand by an uninformed plaintiff, x_p^* , can be shown to be implicitly defined by the equation $1 - F_{Q_d}(J_d x_p^* + c_d) = K f_{Q_d}(J_d x_p^* + c_d)$, where $K \equiv (c_p + c_d)/J_d$ is the ratio of total litigation costs to the defendant's judgment cost.

⁸Reductions in the defendant's belief correspond to more optimistic situations from the defendant's point of view. Bebchuk model cases with very pessimistic defendants will be settled, while those with very optimistic defendants will be settled. The opposite characteristics are true with informed plaintiffs.

⁹With informed plaintiffs the litigated set would be the set of all cases above a horizontal frontier drawn in (q_d, q_p) -space.

Table 1: Taxonomy of Structural Litigation Models According to Reduced Form Elements

| | Litigation rule | Beliefs | Conditional win rate |
|--------------------|---|---|----------------------|
| Bebchuk Screening | Binary/Non-LPG Unilaterally increasing in party optimism Belief-dependent | Continuously distributed Unilaterally degenerate Bilaterally conditionally mean consistent | Unilateral |
| Shavell Screening | Binary/Non-LPG Unilaterally increasing in party optimism Belief-dependent | Discretely distributed Unilaterally degenerate Bilaterally conditionally mean consistent | Unilateral |
| LK Signaling | Non-binary Unilaterally increasing in party optimism Belief-dependent | Continuously distributed Unilaterally degenerate Bilaterally conditionally mean consistent | Unilateral |
| PK-Simulated | Binary/LPG Bilaterally increasing in party optimism Belief-independent | Continuously distributed Bilaterally non-degenerate Bilaterally not conditionally mean consistent Bilaterally not correct on average | Bilateral |
| Friedman & Wittman | Binary/Chatterjee-Samuelson/LPG Bilaterally increasing in party optimism Belief-independent | Continuously distributed Bilaterally non-degenerate Bilaterally not conditionally mean consistent Bilaterally correct beliefs on average | Bilateral |

2.1.2 The Shavell two-point screening model

Shavell (1996) introduced a discrete, two-point version of this model. In Shavell’s version, type L disputes would be won by plaintiffs with probability q^L , and type H would be won by plaintiffs with probability $q^H > q^L$. Suppose defendants are the informed parties. Then $q_d \in \{q^L, q^H\}$, and, with ϕ being the fraction of type L cases, $\bar{q} = \phi q^L + (1 - \phi)q^H$. Thus the joint distribution of party beliefs is given by the probability function

$$f_{Q_d Q_p}(q_d, q_p) = \begin{cases} \phi, & \text{if } q_d = q^L \text{ and } q_p = \bar{q} \\ 1 - \phi, & \text{if } q_d = q^H \text{ and } q_p = \bar{q} \\ 0, & \text{otherwise.} \end{cases}$$

Because the defendant’s belief is correct by assumption, the conditional win rate function satisfies $w(q_d, q_p) = q_d$, as above; it is easy to see that beliefs satisfy conditional mean consistency in this model. Finally, it can be shown that plaintiffs will demand a settlement payment that just equals the expected total loss in litigation for the type H defendant, so that the plaintiff extracts all the settlement surplus with that type; type L defendants will reject this offer and litigate. This implies a binary litigation rule with $L(q^H, \bar{q}) = 0$ and $L(q^L, \bar{q}) = 1$. Consequently, the plaintiff’s win rate among litigated cases is q^L .

2.1.3 The KL signaling model

In the Klerman & Lee (2014) (“KL”) signaling model.¹⁰ The joint density of party beliefs and the conditional win rate function are given in (1) and (2), so only the litigation rule remains to be characterized.¹¹ The LK signaling model differs from Bebchuk’s in that it is the informed party who makes the settlement offer or demand. In the unique class of separating equilibria with informed defendants, KL’s results imply that the probability a case is litigated is

¹⁰This model is an adaptation of the Reinganum & Wilde (1985) signaling model. In the Reinganum and Wilde model, both parties in each case know the true probability that the plaintiff would win, but there is asymmetric information concerning the stakes: the informed side knows the true stakes, while the uninformed side knows only the distribution of stakes. Since the focus in both KL and here is on win rates (RW were interested in other issues), it is necessary to allow variation in the probability the plaintiff would win. In discussing this model, I follow KL in assuming there is no variation in stakes.

¹¹There is a subtlety involved in defining the uninformed party’s subjective belief. In a fully separating equilibrium such as the one that KL describe, the uninformed party can use the informed party’s settlement proposal to identify the informed party’s type. Thus, the uninformed party’s “ex post” belief—her belief after the informed party’s settlement proposal is known to both parties—must be the case’s type, which is the informed party’s subjective belief, too. My focus will be on the uninformed party’s ex ante belief, before she knows the informed party’s offer or demand. This is the proper focus for present purposes, since ex post there is no asymmetric information left to characterize.

$$L(q_d, q_p) = 1 - \exp\left(-\frac{1 - q_d}{K}\right),$$

so that there is probability 0 of litigation when defendants are certain to win, but otherwise the probability of litigation is between 0 and 1.¹² Thus the litigation rule is not binary in the LK signaling model.

2.2 The Simulated Model in Priest and Klein (1984)

In the PK model, there is a random variable Y that cardinally represents true case quality. The plaintiff would actually win a case if it were litigated any time that true case quality at least meets the decision standard y^* , i.e., when $Y \geq y^*$; the defendant would win otherwise. Each party receives a signal of true case quality.¹³ The plaintiff's signal is $Y_p \equiv Y + \epsilon_p$, and the defendant's is $Y_d \equiv Y + \epsilon_d$.

In the model Priest and Klein simulated, the random variable $(\epsilon_d, \epsilon_p, Y)$ has a multivariate normal distribution with mean 0 and variance matrix¹⁴

$$\Sigma \equiv \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

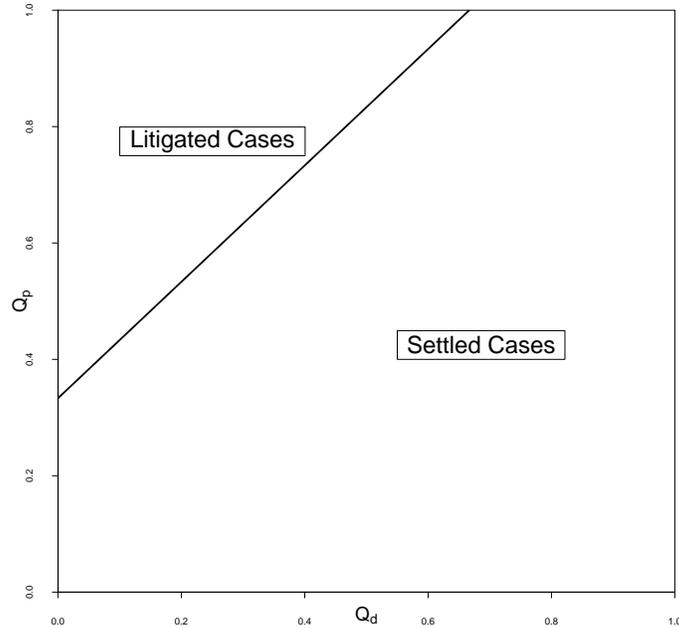
Priest and Klein assume that cases are litigated if and only if it is impossible for both parties to gain from a settlement, which is the LPG litigation rule. Let c_p and c_d be the plaintiff's and defendant's respective costs of litigating defined above, and let s_p and s_d be their costs of negotiating a settlement. Priest and Klein allowed for asymmetric stakes, with J_d being the defendant's expected judgment costs when the plaintiff wins and $J_p \equiv \alpha J_d$ being the plaintiff's expected judgment benefits. When $\alpha=1$, we have symmetric stakes, when $\alpha > 1$ the plaintiff has greater stakes, and when $\alpha < 1$ the defendant has greater stakes. The quantity $q_p \alpha J - c_p$ is the plaintiff's expected gain from litigating. The defendant's expected cost from litigating is $q_d J + c_d$. The gross surplus available from avoiding litigation is the defendant's expected cost less the plaintiff's expected gain: $[q_d J + c_d] - [q_p \alpha J - c_p]$. The net surplus from settlement is positive if this gross surplus exceeds total settlement costs,

¹²KL's main paper discusses the signaling model with informed plaintiffs only; they discuss the case of informed defendants in their online appendix, available for download at <http://www.journals.uchicago.edu/doi/suppl/10.1086/678236>.

¹³Priest and Klein referred to Y_d and Y_p as "estimates," though nothing important turns on the choice of nomenclature.

¹⁴Lee & Klerman (2015) consider variations on the PK model that weaken the joint-normality assumption on the random vector $(\epsilon_p, \epsilon_d, Y)$. For simplicity, I stick to the concrete assumption that Priest and Klein make.

Figure 1: **The LPG Litigation Frontier**



$s_p + s_d$. After a little algebra, we arrive at the LPG litigation rule's necessary and sufficient condition for litigation to occur:

$$q_p \geq \alpha^{-1}[q_d + K^{PK}], \quad (3)$$

where $K^{PK} \equiv (c_d + c_p - s_d - s_p)/J$ is the share of the defendant's stakes that are accounted for by the net of total litigation costs over total settlement costs.

We can represent the litigation/settlement result in the Priest-Klein model via Figure 1, which shows that all cases whose value of (q_d, q_p) lies above the line where (3) holds with equality are litigated, and all other cases are settled; for this reason, I call this line the LPG litigation frontier.¹⁵ The LPG litigation rule is thus binary. Because the location of the LPG litigation frontier in (q_d, q_p) space depends only on α and K^{PK} , the LPG litigation rule is also belief-independent. Finally, it is also bilaterally increasing in party optimism over at least a part of its domain.¹⁶

¹⁵A full analysis of the LPG litigation frontier is unnecessary, but inspection of (3) shows that the frontier's vertical intercept is K^{PK}/α , while its slope is α^{-1} .

¹⁶To see this, observe first that there are points in (q_d, q_p) space such that cases at those points would be settled, while a sufficient move in one party's optimistic direction would cause the case to be litigated; second, there are no points where a move in one party's optimistic direction would switch a case from being litigated to being settled.

Priest and Klein assume that, conditional on the event that the parties' signals are $Y_p=y_p$ and $Y_d=y_d$, their subjective beliefs about the plaintiff's win probability in the event of litigation are

$$q_p = \Phi\left(\frac{y_p - y^*}{\sigma}\right) \quad \text{and} \quad q_d = \Phi\left(\frac{y_d - y^*}{\sigma}\right), \quad (4)$$

where Φ is the *cdf* of the standard normal distribution. Because the parties' signals are continuously distributed, the joint distribution of party beliefs is continuous in the simulated Priest-Klein model. It is also bilaterally non-degenerate, since both parties' beliefs vary. Further, as Table 1 notes, these beliefs do not satisfy conditional mean consistency (see also Section 3 of Lee & Klerman (2014) for a discussion).¹⁷ The beliefs embodied in (4) can still be part of a reduced form litigation model, however, and I shall take them as given for purposes of discussing Priest and Klein's simulated model.

Because true case quality Y is a component of both the parties' signals, the random variables Y_p and Y_d will be positively dependent across cases, even though the idiosyncratic components ϵ_p and ϵ_d are assumed to be independent. I show in A that the joint density of party beliefs in the simulated PK model may be written

$$f_{Q_d Q_p}(q_d, q_p) = A(y^*, \sigma) \exp\left[\frac{-1}{2(2 + \sigma^2)} \left((1 + \sigma^2)[\Phi^{-2}(q_d) + \Phi^{-1}(q_p)^2] \right. \right. \\ \left. \left. - 2\Phi^{-1}(q_d)\Phi^{-1}(q_p) \right. \right. \\ \left. \left. + 2y^*\sigma[\Phi^{-1}(q_d) + \Phi^{-1}(q_p)] \right)\right], \quad (5)$$

where the function A is defined in A and does not depend on party beliefs. I also show in this appendix that the conditional win rate function in the model that Priest and Klein simulated is

$$w_{PK}(q_d, q_p) = \Phi\left(\frac{\Phi^{-1}(q_p) + \Phi^{-1}(q_d) - \sigma y^*}{\sqrt{2 + \sigma^2}}\right). \quad (6)$$

Since w_{PK} depends non-trivially on both q_d and q_p , this conditional win rate function is bilaterally related to party beliefs. Notice also that when $\sigma y^*=0$, the joint density of party beliefs and the conditional win rate function are independent of the legal standard, y^* . As I discuss in a companion paper, this fact helps explain numerous aspects of the relationship between the plaintiff's win rate and the 50% mark in the model that Priest and Klein simulated.¹⁸

¹⁷Nor do these beliefs satisfy the weaker condition of being correct on average, discussed below in conjunction with Friedman & Wittman's (2007) basic litigation game.

¹⁸See Gelbach (2016).

2.3 Friedman & Wittman's (2007) Two-Sided Private Information Model

Now I translate the “basic litigation game” that is the core of Friedman & Wittman's (2007) model of litigation involving two-sided private information. In the basic litigation game, the plaintiff and defendant each receive a signal, respectively denoted θ_p and θ_d , of the probability that the plaintiff will win in the event of litigation. The signals are independent, and each signal has a uniform distribution on the interval $[0, 1]$. In the event of litigation, each party incurs a cost of $c \geq 0$. The amount of the judgment paid to the plaintiff by the defendant in the event the plaintiff wins is normalized to 1. The probability that the plaintiff wins in the event of litigation is $\frac{1}{2}[\theta_p + \theta_d]$.¹⁹

In my notation, the parties' subjective beliefs are simply their signals: $Q_p = \theta_p$ and $Q_d = \theta_d$, so the reduced form conditional win rate function is $w(Q_d, Q_p) = \frac{1}{2}[Q_d + Q_p]$. Since the signals are distributed uniformly and independently on $[0, 1]$, the reduced form joint density of party beliefs is

$$f_{Q_d Q_p}(q_d, q_p) = 1. \quad (7)$$

It can be shown that beliefs in the Friedman & Wittman basic litigation game do not satisfy conditional belief consistency.²⁰ However, these beliefs do satisfy a weaker condition, which I call correct beliefs on average: Party i has correct beliefs on average if $E[Q_i] = E_{Q_d Q_p}[w(Q_d, Q_p)]$.²¹

Friedman & Wittman assume that settlement bargaining occurs according to the Chatterjee & Samuelson (1983, CS) mechanism. Each party makes a sealed settlement offer. If the defendant's offer is at least as great as the plaintiff's then the case is settled at their midpoint. The case goes to trial if the plaintiff's offer is greater than the defendant's. Friedman & Wittman focus on a class of equilibria in which settlement offer functions satisfy a particular symmetry condition. Within this class, the interesting case occurs when $c \in (0, \frac{1}{3})$, with cases litigated whenever²²

¹⁹This is the interpretation provided by Friedman & Wittman at page 108 and page 114. Much of their paper proceeds under the assumption that the plaintiff receives a payment of $\frac{1}{2}[\theta_p + \theta_d]$ with certainty in the event of litigation; given risk neutrality on the part of the parties, there is no important distinction between an award of 1 with probability $\frac{1}{2}[\theta_p + \theta_d]$ and an award of $\frac{1}{2}[\theta_p + \theta_d]$. However, only the former interpretation admits a reduced form as I have defined it.

²⁰To see this, observe that $E[w(Q_d, Q_p)|Q_d = q_d] = \frac{1}{2}[q_d + E(Q_p|Q_d = q_d)] = \frac{1}{2}q_d + \frac{1}{4}$ (with the second equality following by the independence of Q_d and Q_p). This expectation equals q_d only in the special case that $q_d = \frac{1}{2}$. The same result holds for plaintiffs' beliefs, so that the joint distribution of party beliefs fails conditional mean consistency.

²¹The marginal belief distributions are uniform on $[0, 1]$, so $\int q_j f_{Q_j}(q_j) dq_j = \frac{1}{2}$ for $j \in \{p, d\}$. Since the joint density of beliefs is bivariate-uniform on $[0, 1]$, we have $\iint w(q_d, q_p) f_{Q_d Q_p}(q_d, q_p) dq_d dq_p = \iint \frac{1}{2}(q_d + q_p) dq_d dq_p = \frac{1}{2}$, proving the parties' beliefs are correct on average.

²²In this model, trials never occur for $c \geq 1/3$; they always occur for $c \leq 0$.

$$q_p > q_d + 6c - 1. \tag{8}$$

Replacing the inequality symbol with an equals sign, we have a standard LPG litigation frontier with $\alpha=1$ (symmetric stakes) and $K=6c - 1 \in (-1, 1)$.²³ This is an interesting result in its own right, because it shows that there is at least one optimizing model whose equilibrium is an LPG litigation rule.²⁴

2.4 Summary

Each of the models discussed above can be viewed as a “structural” model. By this term I mean a model that sets forth enough information about parties’ costs, stakes, and behavior to derive the relationship between party beliefs and the probability that each case will be litigated or settled, as well as the probability that the case would be won by plaintiffs if actually litigated. The structural models discussed here are quite different in many ways. Yet I have shown that my reduced form approach provides a unified framework to understand all these models in common terms. Table 1 shows that distinct-looking models have considerable reduced form overlap. For example, the reduced forms of the Bebchuk screening and LK signaling models differ only in terms of whether the litigation rule is binary. The Priest-Klein and Friedman & Wittman reduced forms differ only because the Friedman & Wittman model has correct marginal beliefs on average, whereas the Priest-Klein model does not.²⁵ The takehome point from this section, then, is that the reduced form approach provides considerable analytical power to understand apparently disparate structural models of litigation.

²³The case when $K \leq 0$ occurs when $c \leq 1/6$; in terms of the LPG litigation rule, this means settlement costs exceed litigation costs. Notice also that when $c \geq 1/3$, $K > 1$, so the LPG litigation rule would imply that no cases are litigated, just as in Friedman & Wittman’s basic litigation game equilibrium. When $c \leq 0$, the right hand side of the LPG litigation rule is never greater than 0, so the LPG litigation rule would imply that all cases are litigated, just as in Friedman & Wittman’s basic litigation game equilibrium.

²⁴Proper appreciation of this result requires care, however. In the FW basic litigation game, the stakes are symmetric and normalized such that $J = 1$. The parties’ total costs in the FW basic litigation game are $2c$, not $6c - 1$, as (8) would indicate. One way to understand the FW basic litigation game, then, is that it is equivalent to a model with an LPG litigation rule in which $\alpha = J = 1$, with the parties’ total litigation costs being $2c$ and their net settlement costs being $1 - 4c$.

²⁵It is possible to re-work the simulated Priest-Klein model so that it satisfies correct beliefs on average, and even conditional belief consistency. The model’s belief inconsistency appears to have been the result of an error in Priest and Klein’s mathematical discussion. In my notation, the error amounts to incorrectly using the marginal distribution F_{ϵ_p} to evaluate the conditional probability $Q_p = P(Y > y^* | Y_p = y_p) = P(\epsilon_p < y_p - y^* | Y_p = y_p)$, and similarly for ϵ_d . Correcting this mistake is all that is necessary to construct a version of the simulated Priest-Klein model that satisfies conditional belief consistency.

3 Any Reduced Form Can Be Represented as a Generalized Priest-Klein Model

Table 1 seems to indicate substantial differences between the simulated Priest-Klein model and other models, most notably the one-sided asymmetric information models. But I show in this section that one can write *any* model of interest as a kind of Priest-Klein model. It will be helpful to provide a formal definition of a Priest-Klein model:

Definition 1 (Priest-Klein model). There are random variables Y , ϵ_d , and ϵ_p , together with a decision standard y^* , and cost and stakes parameters c_d , c_p , s_d , s_p , J_d , and J_p such that:

1. The plaintiff in a dispute would win in the event of litigation whenever $Y > y^*$.
2. It is common knowledge that the defendant and plaintiff in a dispute know each other's costs and stakes.
3. The defendant observes the case-quality signal $Y_d = Y + \epsilon_d$ and the plaintiff observes the case-quality signal $Y_p = Y + \epsilon_p$, with $E[\epsilon_d] = E[\epsilon_p] = 0$.
4. Given party j 's observed information, $j \in \{d, p\}$, that party has some belief $Q_j \in [0, 1]$ concerning the probability that the plaintiff would win in litigation.
5. The LPG litigation rule holds, so that parties litigate if $Q_p > \alpha^{-1}[Q_d + K^{PK}]$ and settle otherwise, where $K^{PK} = (c_d + c_p - s_d - s_p)/J_d$ and $\alpha = J_d/J_p$.

So far I have taken as given that the stake-asymmetry parameter α and the wedge parameter K^{PK} are fixed. However, in the real world these parameters certainly vary across cases. To accommodate that fact, I shall offer a definition of a generalized Priest-Klein model:

Definition 2 (Generalized Priest-Klein model). A generalized Priest-Klein model is a Priest-Klein model in which K^{PK} and α are allowed to vary, with no restrictions placed on the joint distribution of the vector of random variables $(Y, \epsilon_d, \epsilon_p, K^{PK}, \alpha)$.

Thus the generalized Priest-Klein model is just the Priest-Klein model with K^{PK} and α allowed to have a distribution rather than being fixed parameters. The set of cases that are litigated and the plaintiff's win rate among litigated cases are then determined by the structure of the joint distribution of the random vector $(Y, \epsilon_d, \epsilon_p, K^{PK}, \alpha)$.

I shall use the Shavell screening model with informed defendants to provide an example to illustrate the capacity of the generalized Priest-Klein model to represent seemingly very different models. The first two rows of Table 2 translate party beliefs about the plaintiff's probability of winning in the Shavell screening model into the party's signal in a generalized Priest-Klein model. Suppose a defendant in Shavell's model believes the plaintiff's probability of winning is q^L . Then the corresponding defendant in the generalized Priest-Klein model

Table 2: Representing the Shavell Screening Model with Informed Defendants Using a Generalized Priest-Klein Model

| Variable/parameter | Shavell screening | Generalized Priest-Klein |
|----------------------|--|--|
| Defendant belief | $Q_d = \begin{cases} q^L, & \text{w.p. } \phi \\ q^H, & \text{w.p. } (1 - \phi) \end{cases}$ | $Q_d = \Phi(Y_d)$ |
| Plaintiff belief | $Q_d = \bar{q} \equiv \phi q^L + (1 - \phi)q^H$ | $Q_p = \Phi(Y_p)$ |
| Defendant signal | — | $Y_d = \begin{cases} \Phi^{-1}(q^L), & \text{w.p. } \phi \\ \Phi^{-1}(q^H), & \text{w.p. } (1 - \phi) \end{cases}$ |
| Plaintiff signal | — | $Y_p = \Phi^{-1}(\bar{q})$ |
| True dispute quality | — | $Y (Y_d, Y_p) \sim N(Y_d, 1)$ |
| Decision standard | — | $y^* = 0$ |
| Costs/stakes | $\{c_d, c_p, J_d\}$ | $\{K^{PK}, \alpha\}$ |

receives the signal $Y_d = \Phi^{-1}(q^L)$; since the defendant's belief under the generalized Priest-Klein model described in Table 2 is $Q_d = \Phi(Y_d)$, this defendant has belief $Q_d = \Phi(Y_d) = q^L$. Similar reasoning shows that the defendant beliefs under the two models coincide when the Shavell screening model defendant believes the plaintiff's probability of winning is q^H , and also that the plaintiff's belief is always \bar{q} under both models. Thus the reduced form joint distribution of party beliefs is the same under the two models.

Since the generalized Priest-Klein model's decision standard is $y^*=0$, the plaintiff will actually win under this model whenever $Y>0$. Because the conditional distribution of Y given the parties' information (Y_d, Y_p) is normal with mean Y_d and variance 1, the actual probability the plaintiff will win is the same as the probability that a standard normal random variable exceeds $1 - \Phi(-Y_d) = \Phi(Y_d) = Q_d$. Thus the generalized Priest-Klein model's reduced form conditional win rate function satisfies $w(Q_d, Q_p) = Q_d$, just as in the Shavell screening model.

We need only choose a distribution for K^{PK} and α that ensure the generalized Priest-Klein model has the same litigation rule as the Shavell screening model. Under Shavell's model, the case with $Q_d = q^L$ is litigated, and the one with $Q_d = q^H$ is not. Since the generalized Priest-Klein model's litigation rule is LPG, and since the plaintiff always has belief \bar{q} , we must find values of K^{PK} and α such that $\alpha\bar{q}$ lies between $q^L + K^{PK}$ and $q^H + K^{PK}$. Choosing $\alpha = 1$ and using the definition of \bar{q} , this requirement can be shown to hold whenever $K^{PK} < (q^H - q^L)(1 - \phi)$ (recall that ϕ is the share of q^L -type cases). Thus it is always possible to choose particular values of α and K^{PK} such that the litigation rules coincide under the Shavell screening model and the generalized Priest-Klein model. This establishes that there exists a generalized Priest-Klein model with the same reduced form as the Shavell screening model.

Remarkably, this result can be extended to any model capacious enough to have a reduced form as I defined it at the beginning of section 2.

Theorem 1. Every reduced form model may be represented as a generalized Priest-Klein model. *Proof:* See B.

Theorem 1 provides another view on what does and doesn't separate the simulation results in Priest & Klein (1984) from other, more contemporary models of litigation. The theorem shows that the core difference isn't about the Priest-Klein approach to representing information, because all other models can be represented that way. In fact, the generalized Priest-Klein structure provides a unifying framework. Relatedly, Theorem 1 shows that the distinctions in observable outcomes between the Priest-Klein and asymmetric information models are not the result of the different litigation rules, since the generalized Priest-Klein model always involves the LPG litigation rule. Economists sometimes look askance at the Priest-Klein model for its lack of a bargaining model in which the litigation/settlement outcome is the result of optimizing behavior in equilibrium. Yet Theorem 1 shows that models that do have such explicit structure can be represented the same way. That result

should make us question whether there is anything truly restrictive in adopting the Priest-Klein model’s underlying informational structure. The restrictions evidently come not from this structure, but rather from implied assumptions on the joint distribution of the random variables Y , ϵ_d , ϵ_p , K^{PK} , and α in the generalized Priest-Klein model.

4 What Can We Learn from Data on the Plaintiff’s Win Rate?

In a classic paper, Shavell (1996) demonstrated that any plaintiff’s win rate is possible. Using the Shavell screening model described in section 2.1, it can be shown that a sufficient condition for a separating equilibrium to exist with informed defendants is²⁶

$$q^H > q^L + K^S \frac{f_L}{1 - f_L}, \quad (9)$$

where $K^S \equiv (c_p + c_d)/J$. For any choice of $q^L \in [0, 1)$ it is possible to find (q^H, K, f_L) , with $q^H \in (0, 1]$, such that the inequality above is satisfied. Since only q^H cases are litigated with informed defendants, any plaintiff’s win rate in $[0, 1)$ is possible among litigated cases. A similar argument can be made to show that with informed plaintiffs, any plaintiff’s win rate in $(0, 1]$ is possible among litigated cases. Thus any plaintiff’s win rate in $[0, 1]$ is possible.

Shavell attacked the notion, emphasized by Priest and Klein, that there is a systematic bias of the plaintiff’s win rate toward 50 percent, which would render win rate data useless as a measure of the state of the law.²⁷ Shavell also suggested that asymmetric information accounts for substantial deviations from the 50 percent mark. Using my reduced form approach, however, it is straightforward to show that Shavell’s any-win-rate result can be obtained in a model that (i) uses the LPG litigation rule, as do Priest and Klein; (ii) has a joint distribution of beliefs with discrete support (as in Shavell screening model) but has equal marginal belief distributions (as in Priest and Klein); and (iii) has a non-unilateral conditional win rate function, i.e., one that does not always equal one party’s belief (also as in Priest and Klein). Further, the result can be shown to hold even as the parties’ information becomes very accurate. Thus, neither the equality of the belief distributions nor the accuracy of party beliefs is responsible for pushing the plaintiff’s rate toward any

²⁶This condition is a slightly rewritten version of equation (6) in Shavell (1996, p. 497); I have left out the credibility condition that $q^L J > c_p$, though imposing this condition does not change any result of interest here.

²⁷I treat the relationship between the plaintiff’s win rate and 50% separately in a companion paper, Gelbach (2016). There I show there is nothing necessary about the parties’ having “very accurate information”; what is generally important in explaining a tendency toward one-half is the satisfaction of balance properties by the elements of the reduced form. It just so happens that as “parties obtain very accurate information about trial outcomes” in the simulated Priest-Klein model, these balance conditions are achieved when the stakes are symmetric.

particular value, even when one uses the LPG litigation rule and a bilateral conditional win rate function, as do Priest and Klein.

To show all this, consider the following reduced form:

1. The litigation rule is the LPG litigation rule, with $K^{PK} \equiv (c_d + c_p - s_d - s_p)/J_d$ and $\alpha \in (K^{PK}, 1 + K^{PK})$, so that a case with perfectly optimistic parties, $q_d = 0$ and $q_p = 1$, will be litigated.
2. The conditional win rate function is a weighted average of parties' subjective beliefs: $w(q_d, q_p) = \theta q_d + (1 - \theta)q_p$, where $\theta \in [0, 1]$.
3. The joint distribution of party beliefs puts support on four points and is given by the discrete probability function

$$f_{Q_d Q_p}(q_d, q_p) = \begin{cases} \rho, & \text{if } (q_d, q_p) = (q^1, q^2) \\ \rho, & \text{if } (q_d, q_p) = (q^2, q^1) \\ \gamma - \rho, & \text{if } (q_d, q_p) = (1, 1) \\ 1 - \gamma - \rho, & \text{if } (q_d, q_p) = (0, 0), \end{cases}$$

where $\gamma \geq \rho > 0$ and $\rho + \gamma < 1$. (Notice that the second belief point is constructed by interchanging the parties' beliefs at the first belief point.)

4. The first belief point is litigated: $q^2 > \alpha^{-1}(q^1 + K)$.

It can be shown that the second belief point will never be litigated.²⁸ Since neither the third nor the fourth belief points will ever be litigated, it follows that the plaintiff's win rate is $W^L(q^1, q^2; \theta) \equiv \theta q^1 + (1 - \theta)q^2$. By varying θ between 0 and 1, we can vary the plaintiff's win rate from q^2 to q^1 . Since the point $(q^1, q^2) = (0, 1)$ is litigated, and no other is, we can choose θ and (q^1, q^2) to achieve any desired value of $W^L(q^1, q^2; \theta) \in [0, 1]$. This establishes the any-win-rate result.

Next, observe that each party has probability ρ of believing the plaintiff's chance of victory is q^2 , probability ρ of believing the plaintiff's chance of victory is q^1 , probability

²⁸This claim is equivalent to $\alpha q^1 < q^2 + K$, or $q^2 > \alpha q^1 - K$. I shall prove that the right hand side of this inequality is less than $\alpha^{-1}(q^1 + K)$. This is clearly true when $\alpha \leq 1$. When $\alpha > 1$, it is true if and only if $\alpha^{-1}(q^1 + K) > \alpha q^1 - K$, which implies $(\alpha^2 - 1)q^1 < (1 + \alpha)K$, which holds if and only if $(\alpha - 1)q^1 < K$. Since $\alpha < 1 + K$ by hypothesis, the left hand side is less than $Kq^1 \leq K$, since $q^1 \leq 1$. Therefore, $\alpha q^1 - K < \alpha^{-1}(q^1 + K) < q^2$, with the second inequality holding since (q^1, q^2) is litigated. We have thus established that $q^1 < \alpha^{-1}[q^2 + K]$, which is necessary and sufficient condition for all cases with $(q_d, q_p) = (q^2, q^1)$ to be litigated, given the LPG litigation rule.

$\gamma - \rho$ of believing the plaintiff is certain to win, and probability $1 - \gamma - \rho$ of believing the plaintiff is certain to lose. Thus the parties have identical belief distributions.²⁹

This result can be pushed further to show that Shavell was incorrect in suggesting that accuracy of party beliefs explains a tendency toward 50%. For any $\rho > 0$, the plaintiff's win rate among litigated cases will be $W^L(q^1, q^2; \theta)$, as above. Now consider a sequence of ρ values that converges to 0. Since $W^L(q^1, q^2; \theta)$ does not depend on ρ , the plaintiff's win rate among litigated cases must be constant along this sequence. Thus the limiting value of the plaintiff's win rate as ρ goes to zero is always $W^L(q^1, q^2; \theta)$, which we have seen can take on any value in $[0, 1]$. Thus even as *both* parties' information becomes perfect, it is possible to observe a sequence of plaintiff's win rate values anywhere in $[0, 1]$ among litigated cases. What makes this result possible is that even though at least one party will be mistaken in all litigated cases, cases for which the parties' beliefs are accurate will not be litigated.

Further, because my argument above proceeded with a fixed value of α anywhere in the interval $(K, 1 + K)$, nothing in this argument involves any condition on symmetry or asymmetry of the stakes. Thus in models with the LPG litigation rule, there is no necessary link between the plaintiff's win rate and either symmetry/asymmetry of stakes or party information. In sum, the observed win rate is unconstrained under the LPG litigation rule: Any plaintiff's win rate may be observed in a model in which the LPG litigation rule applies, even if the parties have identical marginal belief distributions, and even along a sequence of party beliefs that converges to perfect accuracy, and regardless of the degree of asymmetry of party stakes.

The results just discussed do not hinge on the use of either the screening model or models using the LPG litigation rule. For example, the results could be generalized to a setting in which a fraction δ of cases involve a discrete version of KL's signaling model in which defendants are informed and make settlement offers, while the remaining $1 - \delta$ share of cases have informed plaintiffs who make settlement offers. It can be shown that the overall observed plaintiff's win rate in this amalgamation of signaling models is $W^L = \delta q^1 + (1 - \delta) q_{pH}$. By varying the fraction δ of informed-defendant cases from 1 to 0, we can obtain any observed plaintiff's win rate between q^1 and q_{pH} . Because q^1 can be as low as 0, while q_{pH} can be as high as 1, this means that any observed plaintiff's win rate in the interval $[0, 1]$ is possible in a signaling model in which some fraction of cases involves informed defendants and some fraction instead involves informed plaintiffs.

Finally as to the LPG litigation rule, the analysis in this section shows that Priest & Klein (1984) were incorrect to suggest that the nonrandom selection of cases for litigation

²⁹One might argue that there is one-sided asymmetric information here in a particular sense, since when $\theta \neq \frac{1}{2}$, one party's belief is systematically more accurate than the other's. While this is true, there is still an important difference by comparison to conventional asymmetric information models such as Shavell's. In those models, the uninformed party would immediately update its beliefs if she were credibly told of the informed party's belief. In the model just described, the parties would be unmoved by any information sharing: it is as if they both know the facts, they both know that the other knows the facts, and they simply disagree about the trial implications of those facts. That is why models like the one I use are often referred to as involving "divergent expectations," or "mutual optimism."

causes at least a bias in the plaintiff’s win rate toward one-half. A bit of algebra shows that in the example just above, the plaintiff’s win rate if all cases were litigated would be

$$W^{\text{ALL}} \equiv \gamma - \rho[1 - (q^1 + q^2)]. \quad (10)$$

Now choose the values of q_{DL} and q^2 so that at the litigated point (q^1, q^2) , we have $q^1 + q^2 = 1$. Then the population plaintiff’s win rate is $W^{\text{ALL}} = \gamma$. Since γ does not affect the plaintiff’s win rate among litigated cases, it will always be possible to choose values of q^1 , q^2 , θ , and γ such that either $W^L(q^1, q^2; \theta) > W^{\text{ALL}} > 0.5$ or $W^L(q^1, q^2; \theta) < W^{\text{ALL}} < 0.5$. Thus, the selection of disputes for litigation may cause a bias either toward or away from one-half, contrary to Priest & Klein’s (1984) argument; further, it is possible for the plaintiff’s win rates among the full population of cases and the subset of litigated cases to be on opposite sides of one-half.³⁰

5 When Can We Rule Out the LPG Litigation Rule Using Observed Data?

The preceding section shows that any value of the plaintiff’s win rate is consistent with a variety of models of litigation. In econometric terms, we might say that the particular model of litigation that applies is unidentified given only data on the plaintiff’s win rate. This is an especially notable result in light of the dominance in the literature of models in which the litigate/settlement decision arises as the result of optimizing behavior by litigants. By contrast, models that rely on the LPG litigation rule typically leave open the question of how the parties arrive at the litigate/settlement result.³¹

Why is the LPG litigation rule unpopular? One issue may be the difficulty of designing a mechanism that induces the parties to reveal their beliefs without outside subsidies. The LPG litigation rule implicitly relies on common knowledge of parties’ beliefs, and if it is not in the parties’ interests to reveal truthfully, then it is not clear how they will know each other’s true beliefs. Consequently, settlement offers and rejections may provide valuable information, so that fully rational parties would behave strategically in providing such information. There is no guarantee that parties will always be able to settle when there is potential surplus from doing so.³²

³⁰See also the discussion at the end of section 4 in Lee & Klerman (forthcoming).

³¹For a model of bargaining without a common prior in an optimizing framework, see, e.g., Yildiz (2003).

³²One way to understand the question here is that it boils down to whether Coasean bargains are feasible. One entry on the negative side is Myerson & Satterthwaite’s (1983) classic result that when two parties with independent valuations bargain over the allocation of a good, incentive-compatible mechanisms cannot be ex post Pareto-efficient without outside subsidies. This result might apply here with the “good” bargained over being conceptualized as the legal claim. However, Myerson and Satterthwaite’s theorem doesn’t apply when valuations are dependent, as they likely are in many litigation contexts due to the zero-sum nature of a court’s award of damages to the plaintiff. McAfee & Reny (1992) show that there are conditions with

While this is a serious drawback of the LPG litigation rule, one should not overlook the fact that more explicit models of the litigation/settlement outcome have their own flaws. First, there are likely few cases in which only one side has private information, and many models do not take this into account. Second, these models place substantial unrealistic structure on the bargaining process. For example, neither the screening nor signaling models discussed here allow the non-offering party to make a counteroffer, even though it would often be irrational for the receiving party not to. For example, in the Shavell screening model, the uninformed party captures all surplus from settlement in those cases that are settled. That leaves the informed party indifferent between litigation and settlement, so countering at a more advantageous settlement amount is at least weakly dominant. Conversely, in the KL signaling model's equilibrium class, the informed party captures all surplus whenever cases are settled. Since the uninformed party can perfectly identify the informed party's belief using the equilibrium settlement offer, a counteroffer weakly dominates the model's equilibrium strategy of randomizing between acceptance and rejection. A third drawback of these models is that they simply assume that one side or the other makes an offer. But the party that makes the offer captures more surplus, so offer-making is valuable. Thus rational parties would compete for that right.

The methodological case against the LPG litigation rule, and in favor of optimizing models that ignore incentives to counteroffer, is thus mixed at best. And LPG-based models are easier to work with than their optimizing counterparts. Further, LPG models are much more intuitive for non-technical readers, due to the simplicity of the no-surplus-no-settlement idea. Finally, there is empirical evidence that people are overly optimistic, which is more in line with the mutual optimism-based LPG litigation rule than its alternatives.³³

Rather than rest on that point, however, in this section I shall offer another reason not to reject the LPG litigation rule: there are broad conditions under which a researcher cannot rule out the possibility that the data were generated by a model that obeys the LPG litigation rule, regardless of the true data generating process. In econometric terms, this section investigates the conditions under which the litigation rule is identified by data, and finds that it often is not.

Theorem 1 establishes that there is a generalized Priest-Klein model that represents any reduced form. Since the litigation rule in the generalized Priest-Klein model is LPG, we can never rule out the LPG litigation rule when Theorem 1 applies. The key to the theorem's proof is the ability to construct an arbitrary conditional distribution of (K, α) , given party beliefs (Q_d, Q_p) . Whenever this ability exists, we will be unable to rule out the LPG litigation rule.

Consider, then, a researcher who knows the stakes to each party as well as their litigation dependent valuations in which *ex post* Pareto-efficiency is possible without net outside subsidies; see Gelbach (2015) for a paper that uses similar results from Cremer & McLean (1988) in the litigation context. While these issues are theoretically interesting, addressing them in detail is beyond the scope of the present paper, and I shall treat the LPG litigation rule as a feasible one for purposes of discussion.

³³See, e.g., Loewenstein, Issacharoff, Camerer & Babcock (1993) and Babcock & Loewenstein (1997).

and settlement costs—information sufficient to determine K and α —as well as the plaintiff’s win rate and the share of disputes that are litigated; I shall refer to this situation as having “full data”. A researcher with full data may sometimes, though not always, be able to rule out the LPG litigation rule if she is willing to assume conditional mean consistency of party beliefs. Given that few data sources will provide full data, this section sketches the outer bounds of the identification question. I answer the question in Theorem 2.

Theorem 2. Fix K^{GPK} and α . Let \bar{L} and \bar{W}_L be the share of litigated cases and the plaintiff’s win rate among these cases. If party beliefs satisfy conditional mean consistency, then the following are alternative sufficient conditions for the LPG litigation rule not to hold:

1. $\frac{K^{GPK}}{\alpha} + \bar{L}(1 - \bar{W}_L) > 1$.
2. $\frac{K^{GPK}}{\alpha} + \frac{\bar{L}\bar{W}_L}{\alpha} > 1$.

Proof: See C.

Thus, conditional mean consistency and the LPG litigation rule are mutually inconsistent when some combination of the following hold: the LPG litigation frontier’s vertical intercept is high, the litigated share is high, and the plaintiff’s win rate among litigated cases is low. It is easy to see that there are feasible parameter values such that inequality (15) is satisfied.³⁴ This shows that it is logically possible for a researcher with full data, together with the assumption of conditional mean consistency, to rule out the applicability of the LPG litigation rule.

On the other hand, the condition for ruling out the LPG litigation rule is actually quite demanding in light of the well known fact that few disputes are actually litigated. Suppose we know that 10% of a set of disputes are litigated to judgment. Given that the litigated share enters (15) multiplicatively with a term that is bounded below 1, the first inequality can be satisfied only if $\frac{K^{GPK}}{\alpha} > 0.9$ —in other words, only if net litigation costs account for at least 90 percent of the plaintiff’s stakes. This seems unlikely for many cases.³⁵

In sum, even the unusually large amount of information embodied by full data, even when married to the strong conditional mean consistency belief characteristic, will likely not enable researchers to rule out the LPG litigation rule much of the time.

6 Are Useful Inferences Possible Using Litigated Cases?

Priest & Klein (1984) is often thought of as associated primarily with hypotheses related to whether the plaintiff’s win rate can be expected to equal or be near 50%. However, the

³⁴For example, set $K^{GPK}/\alpha=0.8$ and $\bar{L}=0.4$; then the inequality is satisfied for all $\bar{W}_L < 0.5$.

³⁵The second inequality suggests that the LPG litigation rule can be ruled out when the plaintiff’s stakes (α) are low, when the net cost share of the defendant’s stakes (K^{GPK}) are high, and when the litigated share (\bar{L}) and the plaintiff’s win rate among litigated cases (\bar{W}_L) are large. Suppose that $\bar{L}\bar{W}_L=0.1$ and suppose that $K^{GPK} = 0.5$, so that net litigation costs are half of the defendant’s stakes. Then the LPG litigation rule can be ruled out only when $\alpha < 0.6$, i.e., only when the plaintiff’s stakes are less than sixty percent of the defendant’s stakes.

true genius of that seminal paper is its simple observation that the systematic selection of disputes for litigation may sever any useful link between the state of the law and observed data on who wins litigated cases. If one is trying to understand the practical import of legal rules, it does not matter whether the plaintiff’s win rate has a tendency toward 50%—but it *does* matter whether legal rules and the plaintiff’s win rate among litigated cases have a discernible relationship.

Recently, Klerman & Lee (2014) (“KL”) have argued that even if the level of the plaintiff’s win rate is not empirically useful, changes in the win rate usefully indicate the direction of changes in the decision standard following statutory or doctrinal innovations. KL prove, for both one-sided asymmetric information models and for the simulated Priest-Klein model, that changes in the plaintiff’s win rate are informative under reasonable conditions.³⁶ They argue that this result justifies the use of win rate data to measure the direction of change in the law. In this section, I argue that their empirical optimism is misplaced.

We can analyze changes in legal rules as operating through two channels. First, changes in legal rules might directly affect the conditional win rate, holding constant the parties’ beliefs: $w(q_d, q_p)$ might increase when the decision threshold y^* moves in a pro-plaintiff direction. The second channel through which changes in legal rules might matter involves the selection of cases for litigation. By changing which cases are litigated and which are settled, changes in legal rules will affect the distribution of $w(q_d, q_p)$ among cases that are litigated.

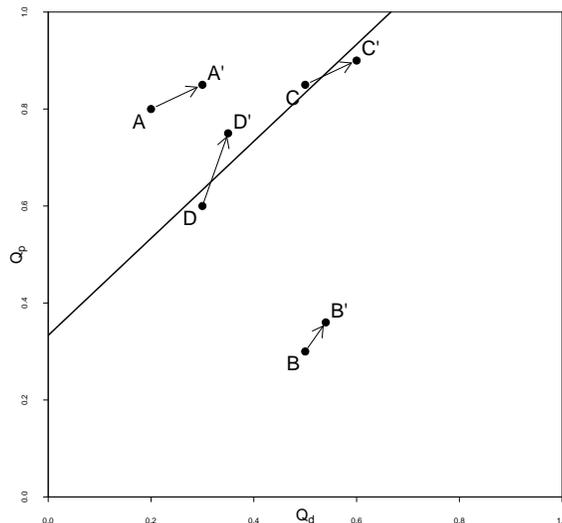
Consider Figure 2. There are initially equal numbers of each of four types of cases, labeled A, B, C, and D. Following a change in legal rules that is perceived to be pro-plaintiff, the parties’ beliefs in these cases shift to points A’, B’, C’, and D’. Each of the “prime” points lies to the northeast of its corresponding initial belief point, reflecting that the parties believe the change in the legal rule increases the plaintiff’s chances of winning in each case.

The upward sloping line in the figure represents the litigation frontier under the LPG litigation rule. The litigation status of A and B cases does not change when the legal rule changes: Type A cases are litigated either way, while type B cases are settled either way. The change in the legal rule does alter the litigation status of type C and type D cases. Type C cases are litigated under the initial legal rule but are settled under the new rule; thus they are “selected out” of litigation by the rule change. Type D cases exhibit the opposite pattern: they are “selected in” to litigation.

The selection-out phenomenon in Figure 2 will cause the plaintiff’s win rate to fall if type C cases have a greater plaintiff’s win rate than type A cases under the initial rule. The selection-in phenomenon will also cause the plaintiff’s win rate to fall if Type D cases have a lower plaintiff’s win rate than type A cases under the initial rule. In this example, then, both forms of case selection might push against the no-selection effect. Depending on whether the direct of selection effect dominates, the plaintiff’s win rate among litigated cases can change

³⁶The most general model that KL consider is more general than the simulated PK model, though it is also less general than my generalized Priest-Klein model; for example, KL’s model imposes independence between true case quality (Y) and the component of each party’s signal that is net of true case quality (ϵ_d and ϵ_p), whereas my generalized Priest-Klein model imposes no structure on this joint distribution.

Figure 2: Selection Effects and a Pro-Plaintiff Shift in Legal Rules



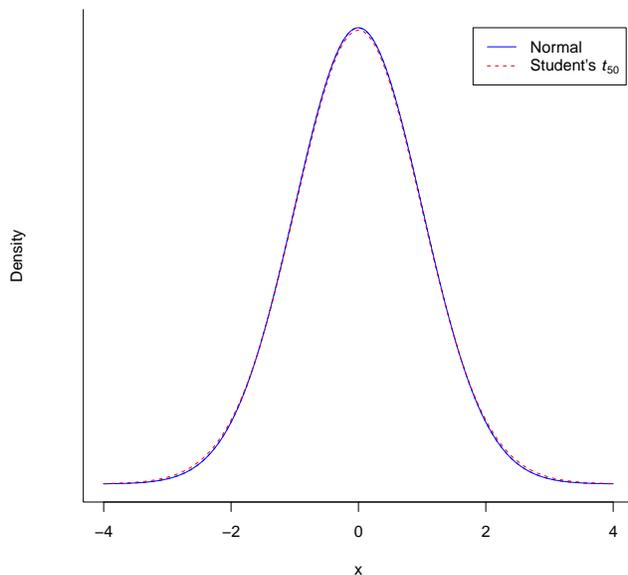
in any direction following a pro-plaintiff change in legal rules.

What should we make, then, of KL’s results that inferences from litigated cases may be informative? To prove their results, KL require certain assumptions. In the case of their version of the Priest-Klein model, KL show that if the density of dispute quality is log-concave, then the plaintiff’s win rate will always increase when the legal standard becomes more pro-plaintiff. Accepting the log-concavity assumption, one would be justified in inferring that the law has moved in plaintiffs’ direction if the plaintiff’s win rate has risen following a change in the law. The reasonableness of the inference thus turns on the reasonableness of the log-concavity assumption.

KL seem to regard the assumption as reasonable, stating in their introduction that “under all standard settlement models and under a wide range of reasonable conditions, one may be able to make valid inferences from the percentage of plaintiff trial victories.” KL, at 230. In the section that discusses their version of the Priest-Klein model, they state that “[t]he class of probability density functions that are log concave is fairly large,” p. 230, and then provide a list of familiar distributions with this property, including the normal distribution that was assumed by Priest and Klein.

But there are also many probability distributions whose densities do not satisfy log-concavity. To pick just one class of examples, no distribution in the Student’s t family has a log-concave density. This is true despite the fact that the Student’s t and normal densities are largely indistinguishable for all but the relatively small values of the Student’s t degrees of freedom parameter. I illustrate this point with Figure 3, which plots the densities for the standard normal distribution and the Student’s t with 50 degrees of freedom over the domain $[-4, 4]$ (which accounts for over 99% of the mass for each density). Despite the similarities of their densities, KL’s sufficient condition for empirical optimism is warranted in

Figure 3: The Standard Normal Density and the Student's t with 50 Degrees of Freedom



the normal case but not the Student's t . Another point is that, in models of the Priest-Klein type, dispute quality is itself purely an abstraction. One can make real-world sense of the statement “that dispute’s quality is greater than the threshold decision standard,” since this statement is tantamount to saying that the plaintiff will win (or has won) in litigation. But what does it mean to say that dispute quality is normally distributed, or has a log-concave density?

Different criticisms can be lodged concerning KL’s results for one-sided asymmetric information models. Suppose the defendant is informed, and ρ is some parameter that tunes the defendant’s marginal belief density f_{Q_d} , and let $f_{Q_d}^L$ be the defendant’s belief density among the set of disputes that are litigated.³⁷ Then if ρ_0 is the original value of the parameter and $\rho_1 > \rho_0$ is the new, more pro-plaintiff value, the plaintiff’s win rate among litigated cases rises whenever

$$\int_0^1 q_d f_{Q_d}^L(q_d; \rho_1) dq_d > \int_0^1 q_d f_{Q_d}^L(q_d; \rho_0) dq_d. \quad (11)$$

Each integral in (11) can be regarded as an expected value of q_d . Thus a sufficient condition for (11) to hold is that when ρ_1 is the parameter value, the density of litigated

³⁷Thus, $f_{Q_d}^L$ is the ratio of f_{Q_d} to its integral from $q_d=0$ to the threshold defendant belief below which all cases are litigated.

cases first-order stochastically dominates the density when ρ_0 is the value. KL assume that changes in legal rules lead to changes in the overall density f_{Q_d} that satisfy the monotone likelihood ratio (“MLR”) property.³⁸ KL are able to prove that the plaintiff’s win rate among litigated cases will rise. The reason the proof works is that the MLR property is sufficient to guarantee first-order stochastic dominance in the two $f_{Q_d}^L$ densities in (11). As KL concede, though, a change in legal rules that simply leads to stochastic dominance in the overall belief distribution—in f_{Q_d} , rather than in $f_{Q_d}^L$ —is by itself not sufficient to yield their result (see page 220 of their paper and Section B of their online appendix).

This is important for two reasons. First, because stochastic dominance in the overall belief distribution f_{Q_d} is all that one can fairly expect from a pro-plaintiff change in the distribution of beliefs. Second, the dominance condition on ϕ_{Q_d} is qualitatively stronger than a first-order dominance relationship for the overall density f_{Q_d} .

We can illustrate the second point with two versions of a simple screening model example, depicted in Figure 4. Defendants are informed, and they have three possible beliefs: that plaintiff’s probability of winning in the event of litigation is q^{low} , q^{med} , or q^{high} , with $q^{low} < q^{med} < q^{high}$. The bottom row of points in each panel of the figure corresponds to an initial distribution in which a third of defendants have each of these three beliefs. The two points with defendant beliefs q^{low} and q^{med} are plotted with solid squares to indicate that cases with these defendant beliefs are litigated; cases with the other defendant belief value are settled, which is indicated by the hollow circle. The diameters of these three points are all equal, representing the fact that the corresponding belief values occur with equal probability. Since litigated cases with defendant beliefs of q^{low} and q^{med} occur in equal proportion, the plaintiff’s win rate among litigated cases is $\bar{q}_0 \equiv \frac{1}{2}(q^{low} + q^{med})$.

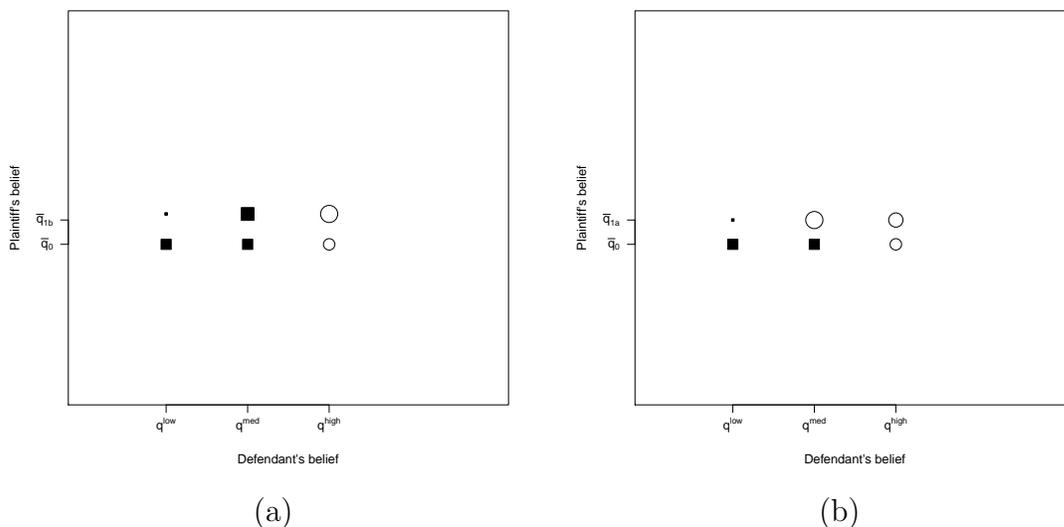
Now consider the top rows of points, which correspond to subsequent distributions following two possible changes in legal rules. Most of the cases in which defendants had belief q^{low} have become cases in which defendants have higher beliefs. In both panels of the figure, only one-twelfth of cases in the top-row distribution have defendant belief q^{low} , with the remaining $(\frac{1}{3} - \frac{1}{12}) = \frac{1}{4}$ probability shifted to either medium or high defendant beliefs. The new distributions are thus unambiguously more pro-plaintiff,³⁹ so plaintiffs have a higher mean belief. This is why the plotted points for the new distributions are higher.

In panel (a), most of the cases in which defendants had low beliefs have become cases in which defendants have high beliefs. This means the change in the distribution of defendant beliefs satisfies the MLR property, so cases with medium defendant beliefs continue to be litigated. As a result, the set of defendant beliefs for which cases are litigated does not change. The only effect of the change in the legal rules on the plaintiff’s win rate is to reduce the frequency of q^{low} cases among those that are litigated while increasing the frequency of q^{med} cases. The end result is that the more pro-plaintiff legal rule is associated with a greater plaintiff’s win rate, just as KL prove.

³⁸That is, they assume that $f_{Q_d}(q_d; \rho)$ is increasing in ρ .

³⁹The new distribution first-order stochastically dominates the original one.

Figure 4: Illustrating How Pro-Plaintiff Changes Can Lead to a Drop in the Plaintiff’s Win Rate Among Litigated Cases in the Screening Model



In panel (b), however, more of the cases in which defendants had low beliefs have become cases in which defendants have *medium* beliefs. The new belief distribution first-order stochastically dominates the old one, but the MLR property is violated. As I have drawn the plot, the violation is substantial enough so that cases with medium defendant beliefs are now settled. Now only cases with low defendant beliefs are litigated. As a result, the plaintiff’s win rate among litigated cases falls to q^{low} . In this example, there is an unambiguous pro-plaintiff shift in legal rules, but the plaintiff’s win rate among litigated cases falls.

What has happened here? With the original legal rule, it made sense for the plaintiff to make a high enough demand that only defendants with belief q^{high} would be willing to settle at that amount. After the change in legal rules, plaintiffs in the panel (b) example are better off reducing their settlement demand so that defendants with belief q^{med} are also willing to settle. The lost settlement payments from defendants with belief q^{high} are made up for by the high probability of obtaining settlement surplus from settling with a defendant having belief q^{med} .⁴⁰

⁴⁰If the plaintiff demands an amount such that she settles with q^{high} defendants only, her respective payoffs when paired against q^{low} , q^{med} , and q^{high} defendants are $q^{low}J - c_p$, $q^{med}J - c_p$, and $q^{high}J + c_d$. If instead the plaintiff demands an amount low enough such that she settles with both q^{med} and q^{high} defendants, her respective payoffs are $q^{low}J - c_p$, $q^{med}J + c_d$, and $q^{high}J + c_d$. Let f_d^{med} and f_d^{high} be the frequencies of the medium and high defendant beliefs in the population. Relative to litigating with defendants holding medium beliefs, plaintiffs who demand an amount that enables them to settle will gain $(c_p + c_d)$ —the total amount that would otherwise be spent litigating the case. On the other hand, plaintiffs facing defendants with high beliefs will have to settle cases for less; this loss is $(q^{high} - q^{med})J$. Accounting for the frequencies of defendant beliefs, the plaintiff will be better off litigating with defendants who hold medium beliefs whenever $f_d^{high} / f_d^{med} > (c_p + c_d) / [(q^{high} - q^{med})J]$. A change in the density that satisfies the MLR property will

The reason the plaintiff’s win rate among litigated cases drops in this example, then, is that the set of cases that are optimal for plaintiffs to settle shrinks: the selection effect outweighs the effect of having a pro-plaintiff change in legal rules. The MLR property is strong enough to foreclose that possibility, while first-order stochastic dominance is not. To be sure, Klerman & Lee (2014, p. 220) do acknowledge this issue. But they treat it as a side point by relegating its discussion to their online appendix. The one qualification they provide in their main text is to state that “our propositions for both the screening and signaling models are phrased in terms of a *sufficiently* more pro-plaintiff legal standard.” Page 220 (emphasis added). But as I have argued, it is hard to distinguish their assumption from the bare assertion that when legal rules change, selection effects are too small to matter. I do not mean to minimize the analytical insight in this work, especially in light of Priest & Klein (1984). But I see little warrant for KL’s empirical optimism, because it is founded on an assumption that has essentially no distance from the desired result.

7 Conclusion

In this paper I have introduced the reduced form approach to litigation models. This approach allows us to focus on common elements of very different-looking litigation models. Numerous insights result. I show that the Priest-Klein framework has much greater flexibility than has previously been understood. Further, I show that it is unlikely that the LPG litigation rule can be rejected empirically, even if researchers have unusually detailed data. Indeed, models relying on the LPG litigation rule are consistent not only with any plaintiff’s win rate, but also with any litigated share of cases, considerably generalizing Shavell’s (1996) famous result in the context of one-sided screening models. Finally, contra Klerman & Lee (2014), the reduced form approach helps us understand how difficult it is to rehabilitate the use of plaintiff win rates as an empirical measure of the law.

Throughout this paper, I have deliberately abstracted from a wide array of real-world procedural details that matter in practice. I have implicitly assumed that cases involve only one claim, one plaintiff, and one defendant, for example. I have also ignored the fact that real-world litigation involves numerous stages in addition to post-trial judgment—most notably, the motion to dismiss, discovery, and summary judgment. The framework set forth here could be generalized to account for each of these important real-world features; I leave this task for future work.

increase the ratio on the left hand side of this inequality. Thus, if the plaintiff’s original optimal settlement demand led to litigation against defendants with medium beliefs, then following a change in legal rules that shifts defendant beliefs in line with the MLR property, it will continue to be optimal for plaintiffs to litigate against defendants holding medium beliefs.

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Appendix A Derivation of the Reduced Form of the Simulated Priest-Klein Model

Define

$$U \equiv \frac{\epsilon_p + Y - y^*}{\sigma} \quad V \equiv \frac{\epsilon_d + Y - y^*}{\sigma} \quad Z \equiv \frac{Y - y^*}{\sigma}$$

The random vector $(U, V, Z)'$ may be written as an affine transformation of $(\epsilon_p, \epsilon_d, Y)'$:

$$\begin{pmatrix} U \\ V \\ Z \end{pmatrix} = -\frac{y^*}{\sigma} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + T \begin{pmatrix} \epsilon_p \\ \epsilon_d \\ Y \end{pmatrix}, \quad \text{with} \quad T \equiv \begin{pmatrix} 1 \\ \frac{1}{\sigma} \end{pmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Under the simulated Priest-Klein model, $(\epsilon_p, \epsilon_d, Y)'$ has a joint normal distribution with mean 0 and a diagonal variance matrix with a 1 in the bottom-right cell and σ^2 in the other two main-diagonal cells. Since any affine transformation of a joint normal random vector is joint normal, $(U, V, Z)'$ is also joint normal. Its mean and variance are⁴¹

$$E \begin{pmatrix} U \\ V \\ Z \end{pmatrix} = -\frac{y^*}{\sigma} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{Var} \begin{pmatrix} U \\ V \\ Z \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sigma^2} \end{pmatrix} \begin{bmatrix} 1 + \sigma^2 & 1 & 1 \\ 1 & 1 + \sigma^2 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

A.1 The conditional win rate function

From the joint normality of (U, V, Z) , the conditional density of z given $(U, V)=(u, v)$ can be shown to be normal with conditional mean and variance

$$\mu_z(u, v) = \frac{1}{2 + \sigma^2} (u + v - \sigma y^*) \quad \text{and} \quad \text{Var}_z(u, v) = \frac{1}{2 + \sigma^2}.$$

⁴¹This follows because $\text{Var}[(U, V, Z)'] = TV(\epsilon_p, \epsilon_d, Y)T'$.

The condition $Y > y^*$ is equivalent to the condition that $Z > 0$, so $P(Y > y^* | Y_d=y_d, Y_p=y_p) = \omega(u, v) \equiv P(Z > 0 | U=u, V=v)$. Since $[Z - \mu_z(u, v)] / \sqrt{\text{Var}_z(u, z)} \sim N(0, 1)$, we have

$$\begin{aligned}\omega(u, v) &= 1 - \Phi\left(-\mu_z(u, v) / \sqrt{\text{Var}_z(u, z)}\right) \\ &= \Phi\left(\mu_z(u, v) / \sqrt{\text{Var}_z(u, z)}\right) \\ &= \Phi\left(\frac{1}{(2 + \sigma^2)^{1/2}} [u + v - \sigma y^*]\right).\end{aligned}$$

Priest and Klein write that the plaintiff's belief about her probability of winning given that she observes signal $Y_p=y_p$ is given by $P(Y \geq 0 | Y_p=y_p)$.⁴² The event $Y \geq y^*$ is equivalent to the event $\epsilon_p \leq Y + \epsilon_p - y^* = Y_p$. Therefore, Priest and Klein calculate, the plaintiff's belief given that $Y_p=y_p$ may be written as $Q_p = F_{\epsilon_p}(y_p - y^*)$. They use similar reasoning to arrive at the conclusion that $Q_d = F_{\epsilon_d}(y_d - y^*)$. These deductions are erroneous. Making Priest and Klein's substitution into $P(Y \geq 0 | Y_p=y_p - y^*)$ yields $P(\epsilon_p \leq y_p - y^* | Y_p=y_p)$. This probability differs from $F_{\epsilon_p}(y_p - y^*)$ because the ϵ_p and Y_p are dependent random variables. Therefore, the conditional distribution of ϵ_p given $Y_p=y_p$ differs from the marginal distribution of ϵ_p , which is what F_{ϵ_p} is. Thus the formulas for party beliefs that Priest and Klein offer at pages 11-12 of their paper are incorrect, and their simulations are based on party beliefs that are inconsistent with parties' full information sets.⁴³ For consistency with Priest and Klein, I shall use their assumed formulas for party beliefs.

Since ϵ_p is normal with mean 0 and variance σ^2 , Priest and Klein's assumptions imply that $F_{\epsilon_p}(y_p) = \Phi(\sigma^{-1}[y_p - y^*])$. Since the event $Y_p=y_p$ is the same as the event $U=u \equiv (y_p - y^*)/\sigma$, the Priest-Klein plaintiff's belief when $Y_p=y_p$ is $\Phi(u)$. Therefore, $U=u$ and $Q_p=q_p \equiv \Phi(u)$ are the same event. Similar analysis shows that $V=v \equiv \sigma^{-1}[y_d - y^*]$ and $Q_d=q_d \equiv \Phi(v)$ are the same event.

Replacing u and v in the function ω with $\Phi^{-1}(q_p)$ and $\Phi^{-1}(q_d)$ then yields

$$\omega(\Phi^{-1}(q_d), \Phi^{-1}(q_p)) = \Phi\left(\frac{1}{(2 + \sigma^2)^{1/2}} [\Phi^{-1}(q_d) + \Phi^{-1}(q_d) - \sigma y^*]\right).$$

Writing $w(q_d, q_p) = \omega(\Phi^{-1}(q_d), \Phi^{-1}(q_p))$ then establishes that the conditional win rate function is as given in the main text.

A.2 The density of litigated cases

After some tedious algebra, (U, V) can be shown to have the density function

$$\begin{aligned}f_{U,V}(u, v) &= A(y^*, \sigma) \times \\ &\exp\left\{-\frac{1}{2(2 + \sigma^2)} [(1 + \sigma^2)(u^2 + v^2) - 2uv + 2y^*\sigma(u + v)]\right\}\end{aligned}$$

⁴²I have translated their notation to my own for clarity.

⁴³See also Section 3 of Lee & Klerman (2014) for a complementary discussion.

where

$$A(y^*, \sigma) \equiv \frac{\sigma}{2\pi\sqrt{2 + \sigma^2}} \exp \left[\frac{-y^{*2}}{2 + \sigma^2} \right] \quad (12)$$

Now change variables from u and v to $q_p = \Phi(u)$ and $q_d = \Phi(v)$. Using the change of variables formula, under which the Jacobian of the transformation is $B(q_d, q_p) \equiv \phi(\Phi^{-1}(q_d))\phi(\Phi^{-1}(q_p))$, it follows that the marginal density function of (Q_d, Q_p) is

$$f_{Q_d Q_p}(q_d, q_p) = \frac{A(y^*, \sigma)}{B(q_d, q_p)} \exp \left\{ -\frac{C(q_d, q_p; y^*, \sigma)}{2(2 + \sigma^2)} \right\}$$

where

$$\begin{aligned} C(q_d, q_p; y^*, \sigma) \equiv & (1 + \sigma^2)(\Phi^{-1}(q_p))^2 + \Phi^{-1}(q_d)^2 \\ & - 2\Phi^{-1}(q_d)\Phi^{-1}(q_p) \\ & + 2y^*\sigma[\Phi^{-1}(q_p) + \Phi^{-1}(q_d)]. \end{aligned}$$

Appendix B Proof of Theorem 1

I shall prove the result in the case of a continuous distribution of party beliefs (Q_d, Q_p) ; the proof for the discrete case is straightforward and similar to the Shavell screening model example in section 4.

Fix y^* to equal any scalar value. Define Y as a random variable whose conditional distribution given $(Q_d, Q_p) = (q_d, q_p)$ is normal with mean $\Phi^{-1}(w(q_d, q_p)) + y^*$ and variance 1. Thus

$$\begin{aligned} P(Y > y^* | Q_d = q_d, Q_p = q_p) &= P[Y - \Phi^{-1}(w(q_d, q_p)) - y^* > \\ & \quad - \Phi^{-1}(w(q_d, q_p)) | Q_d = q_d, Q_p = q_p] \\ &= w(q_d, q_p), \end{aligned}$$

since $Y - \Phi^{-1}(w(q_d, q_p)) - y^*$ has a standard normal distribution conditional on $Q_d = q_d$ and $Q_p = q_p$. This establishes that the conditional probability that the random variable Y exceeds y^* equals the value of the conditional win rate function given the parties beliefs about the plaintiff's probability of winning in litigation.

Now let Y_d be a random variable that is fixed conditional on Q_d , with $Y_d | Q_d = \Phi^{-1}(Q_d) + \lambda_d$, where $\lambda_d \equiv E[Y] - E[\Phi^{-1}(Q_d)]$, and similarly let Y_p be a random variable that is fixed conditional on Q_p , with $Y_p | Q_p = \Phi^{-1}(Q_p) + \lambda_p$, with $\lambda_p \equiv E[Y] - E[\Phi^{-1}(Q_p)]$ and the expectations in the lambdas are taken with respect to the marginal distributions of Y , Q_d , and Q_p . By construction, Y_j has a one-to-one relationship with Q_j , so conditioning on Y_j and conditioning on Q_j involve the same information. It is clear that with knowledge of Y_d and Y_p we may write $Q_d | Y_d = \Phi(Y_d - \lambda_d)$ and $Q_p | Y_p = \Phi(Y_p - \lambda_p)$, so using the change of variables formula the joint density of (Y_d, Y_p) is $f_{Y_d Y_p}(y_d, y_p) = f_{Q_d Q_p}(\Phi(y_d - \lambda_d), \Phi(y_p - \lambda_p))\phi(\Phi(y_d - \lambda_d))\phi(\Phi(y_p - \lambda_p))$.

The joint distribution of (Y, Y_d, Y_p) is then given by $f_{Y|Y_d, Y_p}(y)f_{Y_d Y_p}(y_d, y_p)$. By construction, the leading conditional density is that of a normal random variable with variance 1 and mean $\Phi^{-1}(w(\Phi(y_d - \lambda_d), \Phi(Y_p - \lambda_p)))$.

Next define $\epsilon_d \equiv Y_d - Y$ and $\epsilon_p \equiv Y_p - Y$. By the law of iterated expectations, $E[\epsilon_d] = E_{Q_d}[E(Y_d|Q_d)] - E[Y]$, which is zero since the first term on the right hand side reduces to $E[\Phi^{-1}(Q_d)] + E[Y] - E[\Phi^{-1}(Q_d)] = E[Y]$. A similar argument shows that $E[\epsilon_p] = 0$. Thus the generalized Priest-Klein model relationship between Y, Y_d, Y_p and the ϵ random variables holds, with the residual components being zero on average.

Recall that because Y_j is a monotonic transformation of Q_j , for $j \in \{p, d\}$, the event that party j 's belief is $Q_p = q_p$ and the event that $Y_j = \Phi^{-1}(q_j) + \lambda_j$ are the same. Thus, conditioning on $(Q_d, Q_p) = (q_d, q_p)$ is the same as conditioning on $(Y_d, Y_p) = (\Phi^{-1}(q_d) + \lambda_d, \Phi^{-1}(q_p) + \lambda_p)$. It follows that $P(Y > y^* | Q_d = q_d, Q_p = q_p) = P[Y > y^* | Y_d = \Phi^{-1}(q_d) + \lambda_d, Y_p = \Phi^{-1}(q_p) + \lambda_p]$, so with (Y, Y_d, Y_p) defined as above, the generalized Priest-Klein model replicates all features of the pattern of plaintiff wins, given party signals (Y_d, Y_p) .

We have now constructed a Priest-Klein model whose joint density of signals (Y_d, Y_p) has a one-to-one relationship with the joint density of party beliefs (Q_d, Q_p) in the reduced form, while maintaining the one-to-one relationship between the conditional win rate function and the event that $Y > y^*$ given (Y_d, Y_p) .

All that is left is to show that it is possible for the distribution of K^{GPK} and α , together with the LPG litigation rule in the generalized Priest-Klein model, to yield the same probability of litigation as the reduced form for given party beliefs. That is, we must confirm that it is possible to find a joint distribution of (K^{GPK}, α) and (Y, Y_d, Y_p) such that the conditional probability of litigating in the resulting generalized Priest-Klein model is the same as $L(q_d, q_p)$ when $Y_d = \Phi^{-1}(q_d) + \lambda_d$ and $Y_p = \Phi^{-1}(q_p) + \lambda_p$.

Because the LPG litigation rule implies that cases will be litigated whenever $K^{GPK} < \alpha Q_p - Q_d$, we can satisfy this condition if we can find a conditional distribution of K^{GPK} given (α, Y_d, Y_p) such that $P(K^{GPK} < \alpha' q_p - q_d | \alpha = \alpha', Y_d = \Phi^{-1}(Q_d) + \lambda_d, Y_p = \Phi^{-1}(Q_p) + \lambda_p) = L(q_d, q_p)$. Then the conditional share of litigated cases in the generalized Priest-Klein model given (α, Q_d, Q_p) will match the share litigated in the reduced form for beliefs (Q_d, Q_p) . This establishes that the just-constructed generalized Priest-Klein model fully represents the reduced form in question.⁴⁴

Appendix C Proof of Theorem 2

Start by assuming that the LPG litigation rule applies, and fix the stakes and the parties' costs, so that K^{GPK} and α are fixed. Let the litigated share among cases with these parameter values be \bar{L} , and let the plaintiff's win rate among litigated cases be \bar{W}_L . The set of plaintiff's beliefs for which cases might be litigated is the set that exceed the vertical

⁴⁴One comment is in order. I made no assumptions on the conditional distribution of α given Q_p and Q_d . It is unnecessary to do so provided that settlement costs are allowed to outweigh litigation costs, because then K^{GPK} can take on values as negative as possible; this ensures that even when αq_p is small by comparison to q_d , cases can still be litigated. However, realism might counsel against the idea that settlement costs frequently dominate litigation costs. One can avoid relying on this possibility by imposing enough dependence on the conditional distribution of α given (Q_d, Q_p) . For example, if α always exceeds q_d/q_p , then $\alpha q_p - q_d > 0$ always holds, so K^{GPK} need never be negative for a case to be litigated.

intercept of the LPG litigation frontier, i.e., $\mathcal{Q}_p^* \equiv [K^{GPK}, 1]$. Let the average plaintiff's belief in cases with plaintiff's belief in this set be \bar{q}_{pL} , and let the average plaintiff's belief among settled cases with plaintiff's beliefs in \mathcal{Q}_p^* be \bar{q}_{pS} . Finally, let the average value of the conditional win rate among these settled and litigated cases be $\bar{W}_S(\mathcal{Q}_p^*)$. Then the average value of the conditional win rate function among all cases with plaintiff's belief in \mathcal{Q}_p^* is

$$\bar{W}(\mathcal{Q}_p^*) \equiv \bar{L}\bar{W}_L + (1 - \bar{L})\bar{W}_S(\mathcal{Q}_p^*). \quad (13)$$

Conditional mean consistency can be satisfied only if $\bar{W}(\mathcal{Q}_p^*) = \bar{q}_{pL}$. Under the LPG litigation rule, plaintiffs in any litigated case must believe that their probability of winning exceeds $\frac{K^{GPK}}{\alpha}$, so $\bar{q}_{pL} \geq \frac{K^{GPK}}{\alpha}$. Thus we can rule out the LPG litigation rule any time the data are inconsistent with a value of \bar{W}_L at least equal to $\frac{K^{GPK}}{\alpha}$. From the definition of \bar{W}_L in (13), this means we can rule out the LPG litigation rule whenever

$$\bar{W}_S(\mathcal{Q}_p^*) < \frac{\frac{K^{GPK}}{\alpha} - \bar{L}\bar{W}_L}{1 - \bar{L}}. \quad (14)$$

By hypothesis, we have data on K^{GPK} , α , \bar{L} , and \bar{W}_L . Since \bar{W}_S is bounded in $[0, 1]$, (14) is guaranteed to hold if its right hand side exceeds 1. That will happen any time

$$\frac{K^{GPK}}{\alpha} + \bar{L}(1 - \bar{W}_L) > 1, \quad (15)$$

which is the first proffered sufficient condition.

To establish that the second proffered condition is sufficient, let $\mathcal{Q}_d^* \equiv [0, \alpha - K^{GPK}]$, which, given K^{GPK} , is the set of defendant's beliefs such that a dispute could possibly be litigated under the LPG litigation rule. Let \bar{q}_{dL} be the average defendant belief among all cases with defendant belief in \mathcal{Q}_d^* , and let $\bar{W}(\mathcal{Q}_d^*)$ be the average of the conditional win rate function over the set of all cases with defendant beliefs in \mathcal{Q}_d^* . Conditional mean consistency requires that $\bar{W}(\mathcal{Q}_d^*) = \bar{q}_{dL}$, so if the LPG litigation rule holds, we must have $\bar{W}(\mathcal{Q}_d^*) < \alpha - K^{GPK}$. Define $\bar{W}_S(\mathcal{Q}_d^*)$ to be the average of the conditional win rate function among settled cases with defendant's belief in \mathcal{Q}_d^* . Then we have

$$\bar{W}(\mathcal{Q}_d^*) \equiv \bar{L}\bar{W}_L + (1 - \bar{L})\bar{W}_S(\mathcal{Q}_d^*), \quad (16)$$

so that the LPG litigation rule can hold only if $\bar{W}_S(\mathcal{Q}_d^*) < [\alpha - K^{GPK} - \bar{L}\bar{W}_L]/(1 - \bar{L})$. Since $\bar{W}_S(\mathcal{Q}_d^*) \geq 0$, the inequality above will be violated whenever its right hand side is negative. Thus, the LPG litigation rule cannot hold if $\frac{K^{GPK} + \bar{L}\bar{W}_L}{\alpha} > 1$, which is the second proffered condition in the theorem.